

2025-2026 学年八年级数学下学期 3 月学情自测

培优卷 · 参考答案

一、选择题：本题共 6 小题，每小题 2 分，共 12 分。在每小题给出的四个选项中，只有一项是符合题目要求的。

题号	1	2	3	4	5	6
答案	B	A	C	C	D	A

二、填空题：本题共 12 小题，每小题 3 分，共 36 分。

7. $(5,6)$; 8. 100° ; 9. ± 1 ; 10. 100; 11. 3; 12. 24; 13. $\sqrt{10}$;

14. 4; 15. $(0,8)$ 或 $(0,-8)$; 16. 3; 17. $(1,-4)$; 18. 4

三、解答题（本题共 7 小题，共 52 分。解答应写出文字说明、证明过程或演算步骤。）

19. (6 分)

【详解】(1) 解：将点 $B(3,0)$ 向上平移 2 个单位长度，再向右平移 1 个单位长度，得到的点 D 的横坐标为 $3+1=4$ ，纵坐标为 $0+2=2$ ，即 $D(4,2)$ ；(3 分)

(2) 设点 P 的坐标为 $(0,y)$ ，则 $PO=|y|$ ，

$$\because AB=3-(-1)=4, \quad S_{\triangle PAB}=2,$$

$$\therefore S_{\triangle PAB}=\frac{1}{2} \times 4 \times |y|=2,$$

$$\therefore |y|=1, \quad (5 \text{ 分})$$

解得 $y=\pm 1$ ，

$\therefore$ 点 P 的坐标为 $(0,1)$ 或 $(0,-1)$ 。(6 分)

20. (6 分)

【详解】证明：(1) $\because$ 点 D, E, F 分别是 AB, BC, CA 的中点， $\therefore DE, EF$ 都是 $\triangle ABC$ 的中位线。
 $\therefore EF \parallel AB, DE \parallel AC, \therefore$ 四边形 $ADEF$ 是平行四边形。(3 分)

(2) $\because$ 四边形 $ADEF$ 是平行四边形， $\therefore \angle DEF=\angle BAC$ 。

$\because D, F$ 分别是 AB, CA 的中点， AH 是边 BC 上的高， $\therefore DH=AD, FH=AF$ 。

$\therefore \angle DAH=\angle DHA, \angle FAH=\angle FHA$ 。

$$\because \angle DAH + \angle FAH = \angle BAC, \quad \angle DHA + \angle FHA = \angle DHF,$$

$$\therefore \angle DHF = \angle BAC. \therefore \angle DHF = \angle DEF. \quad (6 \text{ 分})$$

21. (6 分)

【详解】(1) 证明: $\because$ 点 E 为 AD 的中点,

$$\therefore AE = DE,$$

$\because$ 四边形 $ABCD$ 是平行四边形,

$$\therefore AB = DC, \quad AB \parallel DC,$$

$$\therefore \angle AFE = \angle DCE,$$

$$\text{又} \because \angle AEF = \angle DEC,$$

$$\therefore \triangle AEF \cong \triangle DEC,$$

$$\therefore AF = DC,$$

$$\therefore AB = AF,$$

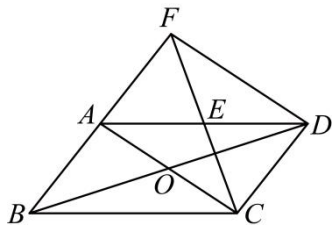
即 A 为 BF 的中点. (3 分)

(2) 解: $AD = 2AB$; (不唯一) (4 分)

理由如下: 由 (1) 得 $AF \parallel DC$, 且 $AF = DC$,

$\therefore$ 四边形 $ACDF$ 是平行四边形,

$\because$ 四边形 $ABCD$ 是平行四边形,



$$\therefore AD = BC,$$

$$\because AD = 2AB,$$

$$\therefore BC = 2AB,$$

$$\because BF = 2AB,$$

$$\therefore BC = BF,$$

$$\text{又} \because \angle ABC = 60^\circ,$$

$\therefore \triangle BCF$ 是等边三角形,

$$\because AB = AF,$$

$$\therefore AC \perp AF,$$

$\because PQ$ 垂直平分 BE ,

$$\therefore PB = PE, \quad QB = QE,$$

$$\therefore \angle PBE = \angle PEB,$$

$$\therefore \angle PBE = \angle EBQ, \text{ 且 } BE \perp PQ,$$

$$\therefore \angle PBO = \angle QBO, BO = BO, \angle BOP = \angle BOQ = 90^\circ,$$

$$\therefore \triangle PBO \cong \triangle QBO (ASA),$$

$$\therefore BP = BQ, \quad (3 \text{ 分})$$

$$\therefore PB = BQ = QE = PE,$$

$\therefore$ 四边形 $BPEQ$ 是菱形; (4 分)

$$(2) \text{ 解: } \because \angle A = 90^\circ, AB = 6, BE = 10,$$

$$\therefore AE = \sqrt{BE^2 - AB^2} = \sqrt{10^2 - 6^2} = 8, \quad (5 \text{ 分})$$

$\because$ 四边形 $BPEQ$ 是菱形,

$$\therefore OP = OQ, OB = OE = \frac{1}{2} BE = 5, BP = PE, \angle BOP = 90^\circ,$$

$$\therefore \text{设 } BP = PE = x, \text{ 则 } AP = AE - PE = 8 - x,$$

$$\text{在 } Rt\triangle ABP \text{ 中, } AP^2 + AB^2 = BP^2,$$

$$\therefore (8-x)^2 + 6^2 = x^2,$$

$$\text{解得, } x = \frac{25}{4},$$

$$\therefore BP = \frac{25}{4}, \quad (7 \text{ 分})$$

$$\text{在 } Rt\triangle BOP \text{ 中, } PO = \sqrt{BP^2 - BO^2} = \sqrt{\left(\frac{25}{4}\right)^2 - 5^2} = \frac{15}{4},$$

$$\therefore PQ = 2PO = 2 \times \frac{15}{4} = \frac{15}{2}. \quad (8 \text{ 分})$$

24. (8 分)

【详解】(1) 证明: $\because DE \perp AD, \quad EG \perp DE,$

$$\therefore AD \parallel EG.$$

$$\because DA = EG,$$

$\therefore$ 四边形 $ADEG$ 是平行四边形.

$$\because DE \perp AD$$

∴ 四边形 $ADEG$ 是平行四边形 (3 分)

(2) 解: ① ∵ 四边形 $OABC$ 为矩形, 点 B 的坐标为 $(6, 4)$,

$$\therefore OA = BC = 6, \quad OC = AB = 4, \quad OA \parallel BC, \quad \angle B = 90^\circ,$$

$$\therefore \angle BDA = \angle OAD, \quad \angle B = \angle EOG = 90^\circ.$$

由 (1) 知, 四边形 $ADEG$ 是矩形,

$$\therefore AD = GE, \quad AD \parallel GE,$$

$$\therefore \angle OAD = \angle OEG,$$

$$\therefore \angle BDA = \angle OEG.$$

在 $\triangle ABD$ 和 $\triangle GOE$ 中,

$$\begin{cases} \angle B = \angle EOG \\ \angle BDA = \angle OEG \\ AD = GE \end{cases}$$

$$\therefore \triangle ABD \cong \triangle GOE \text{ (AAS)},$$

$$\therefore OG = AB = 4. \quad (6 \text{ 分})$$

$$\text{② 由 (2) ① 知, } OE = BD = \frac{8}{3},$$

$$\therefore AE = OE + OA = \frac{8}{3} + 6 = \frac{26}{3}.$$

∵ 四边形 $ADEG$ 为矩形, 对角线 AE , DG 交于点 H ,

$$\therefore EH = \frac{1}{2}AE = \frac{13}{3},$$

$$\therefore OH = EH - OE = \frac{13}{3} - \frac{8}{3} = \frac{5}{3},$$

$$\therefore \text{点 } H \text{ 的坐标为 } \left(\frac{5}{3}, 0\right). \quad (8 \text{ 分})$$

25. (10 分)

【详解】(1) 由 $AE \perp DF$ 证明 $AE = DF$

$$\therefore AE \perp DF$$

$$\therefore \angle EAD + \angle ADH = 90^\circ$$

∵ 四边形 $ABCD$ 为正方形

$$\therefore AD = CD, \quad \angle ADC = \angle BCD = 90^\circ$$

$$\therefore \angle CDF + \angle ADH = 90^\circ$$

$$\therefore \angle CDF = \angle DAE$$

$$\therefore \triangle CDF \cong \triangle DAE$$

$$\therefore AE = DF \quad (3 \text{ 分})$$

(2) ①解: 由翻折可得: $\triangle ADE \cong \triangle AD'E$,

$$\therefore AD = AD', DE = D'E,$$

$$\therefore AE \perp DD',$$

即 $AE \perp DF$

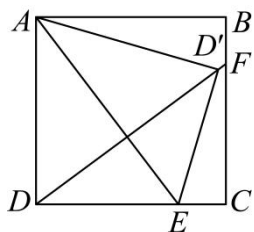
$$\therefore \angle ADF + \angle DAE = 90^\circ,$$

$$\because AB = 4, DE = 3,$$

$$\therefore AE = \sqrt{AD^2 + DE^2} = \sqrt{4^2 + 3^2} = 5,$$

$\because$ 由问题 (1) 可知 $AE = DF$,

$$\therefore DF = AE = 5; \quad (6 \text{ 分})$$



②证明: $\because BG \parallel D'E$,

$$\therefore \angle AED' = \angle AGB,$$

$\because$ 正方形 $ABCD$ 中 $AB \parallel CD$, $AD = AB$,

$$\therefore \angle BAE = \angle AED,$$

$\because$ 由翻折可知: $\angle AED = \angle AED'$, $\angle ADD' = \angle AD'D$,

$$\therefore \angle BAE = \angle AGB,$$

$$\therefore BA = BG,$$

$$\therefore \angle BAG = \angle BGA,$$

$$\because \angle FDC = \angle DAE,$$

$$\therefore 90^\circ - \angle DAE = 90^\circ - \angle CDF,$$

$$\therefore \angle ADD' = \angle DEA,$$

$$\therefore \angle ADD' = \angle BAG,$$

$$\therefore \angle ADD' = \angle BAG = \angle AD'D = \angle BGA,$$

$$\therefore \triangle ADD' \cong \triangle BAG \quad (AAS); \quad (10 \text{ 分})$$

