

2024-2025 学年八年级数学下学期期末模拟卷

一、选择题（本大题共 6 小题，每小题 3 分，满分 18 分。在每个小题给出的四个选项中，只有一项符合题目要求的）

1	2	3	4	5	6
C	C	A	A	C	D

二、填空题（本大题共 12 小题，每小题 2 分，满分 24 分）

7. 10 8. $x = -2$ 9. $y^2 - 2y + 1 = 0$ 10. 10

11. $x > 2$ 12. $y = -3x - 6$ 13. $\frac{1}{4}$ 14. 20

15. $m < \frac{7}{4}$ 或 $m = 2$ 16. 56 17. $9\sqrt{3} \leq S \leq 18$ 18. $\sqrt{3} - 1$

三.解答题（本大题共 8 小题，58 分。解答应写出文字说明，证明过程或演算步骤）

19. （4 分）【详解】解: $\begin{cases} x + 2y = 4 \text{①} \\ x^2 - xy - 2y^2 = 0 \text{②} \end{cases}$,

将②变形可得 $(x - 2y)(x + y) = 0$, (1 分)

即 $x - 2y = 0$ 或 $x + y = 0$, (1 分)

故方程组可变形得 $\begin{cases} x + 2y = 4 \\ x - 2y = 0 \end{cases}$ 或 $\begin{cases} x + 2y = 4 \\ x + y = 0 \end{cases}$, (1 分)

解得 $\begin{cases} x = 2 \\ y = 1 \end{cases}$ 或 $\begin{cases} x = -4 \\ y = 4 \end{cases}$,

故原方程组的解为 $\begin{cases} x = 2 \\ y = 1 \end{cases}$ 或 $\begin{cases} x = -4 \\ y = 4 \end{cases}$ (1 分)

20. （6 分）【详解】（1）证明: $\because$ 四边形 $ABCD$ 是平行四边形,

$\therefore AB = CD, AD = BC, \angle B = \angle D, \angle A = \angle C,$

$\because BE = DG, BF = DH, \text{ 且 } \angle B = \angle D,$

$\therefore \triangle BEF \cong \triangle DGH (SAS),$

$\therefore EF = HG,$

同理可得 $EH = FG,$

∴ 四边形 $EFGH$ 是平行四边形；…… (3 分)

(2) 四边形 $EFGH$ 是矩形，证明如下，

$$\because AB = BC, BE = BF$$

$$\therefore AB = BC = CD = AD, BE = BF = DH = DG,$$

$$\therefore AE = AH,$$

$$\because AD \parallel BC,$$

$$\therefore \angle B + \angle A = 180^\circ,$$

$$\because BE = BF, AE = AH,$$

$$\therefore \angle BEF = \angle BFE = \frac{180^\circ - \angle B}{2}, \quad \angle AEH = \angle AHE = \frac{180^\circ - \angle A}{2},$$

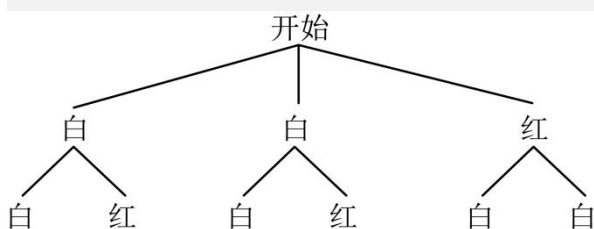
$$\therefore \angle AEH + \angle BEF = 90^\circ,$$

$$\therefore \angle FEH = 90^\circ,$$

∴ 平行四边形 $EFGH$ 是矩形。…… (3 分)

21. (6 分) 【详解】(1) 解：摸出的球是白球的概率是 $2 \div (2+1) = \frac{2}{3}$ ；…… (2 分)

(2) 解：画树状图为：



共有 6 种等可能的结果数，其中两次摸出的球都是白球的结果数为 2，

即两次都是摸出白球的概率为： $\frac{2}{6} = \frac{1}{3}$ ；…… (2 分)

(3) 解：设往箱子里放红球 x 个，白球 1 个，根据题意得：

$$\frac{2+1}{3+1+x} = \frac{3}{5}, \quad \text{即 } 15 = 12 + 3x$$

解得： $x = 1$ ，

经检验 $x = 1$ ，是原方程的解，

∴ 往箱子里放红球 1 个，白球 1 个，摸出白球的概率为 $\frac{3}{5}$ …… (2 分)

22. (6 分) 【详解】(1) ∵ 四边形 $ABCD$ 是平行四边形，

$$\therefore AB \parallel CD, AB = CD,$$

$$\because EF \parallel AD,$$

$\therefore$ 四边形 $ADFE$ 是平行四边形,

$\therefore AE = DF$,

$\because$ 点 E 是 AB 的中点,

$$\therefore AE = BE = \frac{1}{2} AB,$$

$$\therefore AE = DF = \frac{1}{2} CD = CF,$$

$\therefore$ 四边形 $AFCE$ 是平行四边形,

$\therefore AF = EC$,

$\therefore \overrightarrow{AF} = \overrightarrow{EC}$,

故答案为: $\overrightarrow{EC}$; $\cdots$ (2 分)

(2) $\because \overrightarrow{AB} = \vec{a}$, 点 E 是 AB 的中点,

$$\therefore \overrightarrow{AE} = \frac{1}{2} \overrightarrow{AB} = \frac{1}{2} \vec{a},$$

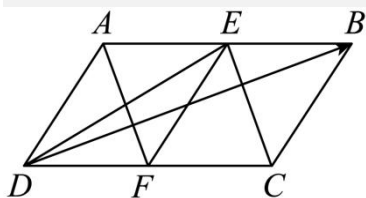
$\because \overrightarrow{AD} = \vec{b}$, 四边形 $ADFE$ 是平行四边形,

$$\therefore \overrightarrow{AF} = \overrightarrow{AE} + \overrightarrow{AD} = \frac{1}{2} \vec{a} + \vec{b}, \overrightarrow{DE} = \overrightarrow{AE} - \overrightarrow{AD} = \frac{1}{2} \vec{a} - \vec{b},$$

故答案为: $\frac{1}{2} \vec{a} + \vec{b}$, $\frac{1}{2} \vec{a} - \vec{b}$; $\cdots$ (2 分)

(3) $\because \overrightarrow{AF} = \overrightarrow{EC}$,

$$\therefore \overrightarrow{DE} + \overrightarrow{AF} - \overrightarrow{BC} = \overrightarrow{DE} + \overrightarrow{EC} - \overrightarrow{BC} = \overrightarrow{DC} + \overrightarrow{CB} = \overrightarrow{DB}$$



$\therefore$ 图中 $\overrightarrow{DB}$ 为所求向量. $\cdots$ (2 分)

23. (6 分) 【详解】(1) 解: 甲车离开 A 地将一批货物送到 B 地对应图像的函数解析式为: $y = \frac{360}{9}x = 40x$,

故答案为: $y = 40x$; $\cdots$ (2 分)

(2) 解: 令 $y = 180$, 则 $40x = 180$,

解得: $x = 4.5$,

故答案为: 4.5; $\cdots$ (2 分)

(3) 解: 乙车的速度为 $\frac{180}{4.5-1.5} = 60$ 千米/时,

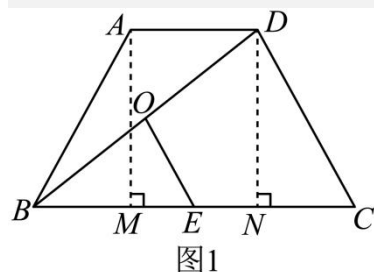
$$\text{则 } 40x + 60(x - 1.5 - 0.5) = 360 \times 2,$$

解得: $x = 8.4$,

$\therefore$ 距离 A 地距离为 $40x = 40 \times 8.4 = 336$ 千米,

故答案为: 336. …… (2 分)

24. (9 分) 【详解】(1) 解: 过 A 作 $AM \perp BC$, 过 D 作 $DN \perp BC$, 垂足分别为 M、N,



则 $\angle AMB = \angle AMC = \angle DNC = 90^\circ$,

$\therefore AM \parallel DN$,

$\therefore AD \parallel MN$,

$\therefore$ 四边形 AMND 是平行四边形,

$\therefore MN = AD = 5$, $AM = DN$,

$\therefore AB = CD$,

$\therefore \text{Rt}\triangle ABM \cong \text{Rt}\triangle DCN (\text{HL})$,

$\therefore BM = CN$,

$\therefore BC = 13$, $MN = 5$,

$\therefore BM = CN = 4$,

在 $\text{Rt}\triangle ABM$ 中,

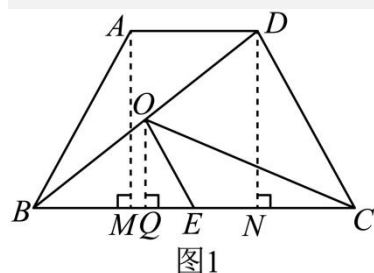
$\therefore \angle ABC = 60^\circ$

$\therefore \angle BAM = 30^\circ$,

$\therefore BM = \frac{1}{2} AB$,

$\therefore AB = 8$. …… (3 分)

(2) 过点 O 作 $OQ \perp BC$, 垂足为点 Q, 则 $\angle OQE = 90^\circ$,



$\because O$ 是 BD 的中点, E 是 BC 的中点,

$$\therefore OE = \frac{1}{2}CD = \frac{1}{2}AB = 4, \quad OE \parallel CD, \quad CE = \frac{1}{2}BC = \frac{13}{2},$$

$$\therefore \angle OEB = \angle C = 60^\circ,$$

$$\therefore \angle QOE = 90^\circ - \angle OEQ = 30^\circ,$$

$$\text{在 Rt}\triangle OEQ \text{ 中, } EQ = \frac{1}{2}OE = 2, \quad OQ = \sqrt{OE^2 - QE^2} = \sqrt{4^2 - 2^2} = 2\sqrt{3},$$

$$S_{\triangle OEC} = \frac{1}{2}EC \cdot OQ = \frac{1}{2} \times \frac{13}{2} \times 2\sqrt{3} = \frac{13}{2}\sqrt{3}. \quad \dots\dots (3 \text{ 分})$$

$$(3) \because AD \parallel BC,$$

$$\therefore \angle DFO = \angle OEB,$$

$\because O$ 是 BD 的中点,

$$\therefore OB = OD,$$

在 $\triangle DOF$ 和 $\triangle BOE$ 中,

$$\begin{cases} \angle DFO = \angle OEB \\ \angle FOD = \angle EOB, \\ OD = OB \end{cases}$$

$$\therefore \triangle DOF \cong \triangle BOE \text{ (AAS)},$$

$$\therefore DF = BE,$$

$$\because DF \parallel BE,$$

$\therefore$ 四边形 $BEDF$ 是平行四边形,

$\because EF$ 平分 $\angle BED$,

$$\therefore \angle BEF = \angle DEF,$$

$$\because \angle BEF = \angle DFE,$$

$$\therefore \angle DEF = \angle DFE,$$

$$\therefore DE = DF,$$

$\therefore$ 四边形 $BEDF$ 是菱形,

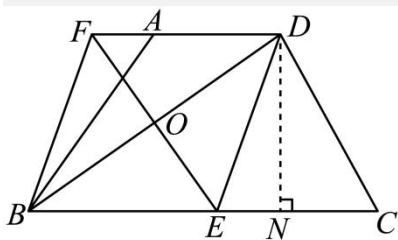


图2

过点 D 作 $DN \perp BC$ 于点 N ,

由 (1) 可知, $CN = 4$,

$$\therefore BN = BC - CN = 13 - 4 = 9,$$

$$\text{由勾股定理得 } DN = \sqrt{CD^2 - CN^2} = \sqrt{AB^2 - CN^2} = \sqrt{8^2 - 4^2} = 4\sqrt{3},$$

设 $DE = BE = x$, 则 $EN = 9 - x$,

$$\text{在 Rt}\triangle END \text{ 中, } EN^2 + DN^2 = DE^2,$$

$$\text{即 } (9-x)^2 + (4\sqrt{3})^2 = x^2,$$

$$\text{解得 } x = \frac{43}{6},$$

$$\therefore \text{四边形 } BEDF \text{ 的周长} = 4DE = 4 \times \frac{43}{6} = \frac{86}{3}. \quad \cdots \cdots (3 \text{ 分})$$

25. (9 分) 【详解】(1) 解: $\because$ 直线 $y = -2x + 4$ 与 y 轴交于点 A , 与 x 轴交于点 B ,

$\therefore$ 当 $x = 0$ 时, $y = 4$; 当 $y = 0$ 时, $x = 2$,

$$\therefore A(0, 4), B(2, 0); \quad \cdots \cdots (3 \text{ 分})$$

$$(2) \because \text{直线 } y = \frac{4}{3}x + 4, \text{ 当 } x = 0 \text{ 时, } y = 4; \text{ 当 } y = 0 \text{ 时, } x = -3,$$

$$\therefore M(-3, 0),$$

$$\therefore BM = 2 - (-3) = 5,$$

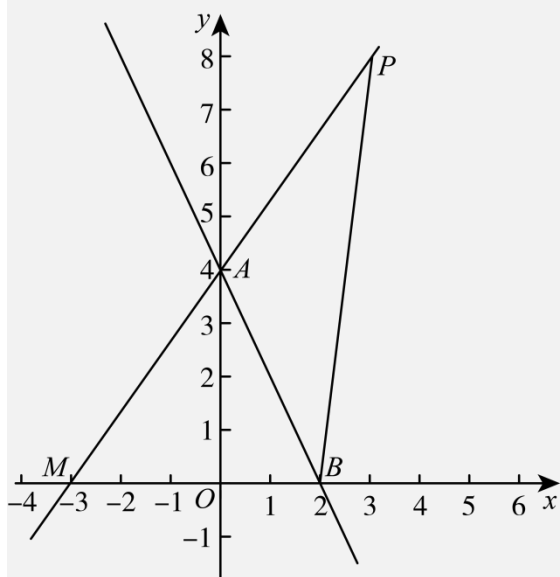
$$\text{设 } P\left(x, \frac{4}{3}x + 4\right),$$

$$\therefore S_{\triangle PAB} = S_{\triangle PBM} - S_{\triangle ABM} = \frac{1}{2} \times 5 \times \left(\frac{4}{3}x + 4\right) - \frac{1}{2} \times 5 \times 4 = 10,$$

解得: $x = 3$,

$$\therefore \frac{4}{3}x + 4 = 8,$$

$$\therefore P(3, 8); \quad \cdots \cdots (3 \text{ 分})$$



(3) 设点 $C(0, y)$,

$$\because AB \perp BC,$$

$$\therefore \angle ABC = 90^\circ,$$

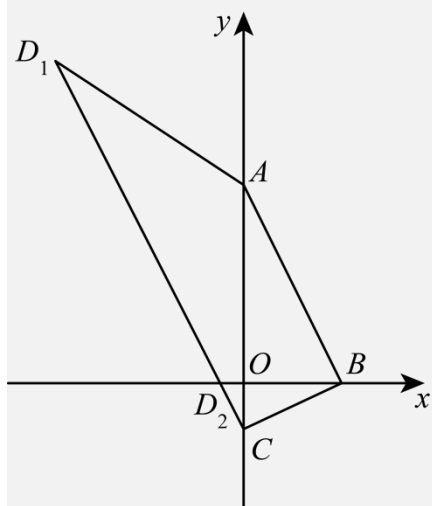
$$\therefore AB^2 + BC^2 = AC^2$$

$$\therefore 20 + 2^2 + y^2 = (4 - y)^2,$$

$$\text{解得: } y = -1,$$

$$\therefore C(0, -1),$$

当 $AB \parallel CD$ 时, 如图所示:



$$\therefore \text{直线 } CD \text{ 的解析式为 } y = -2x + 1,$$

$$\text{设点 } D(k, -2k - 1),$$

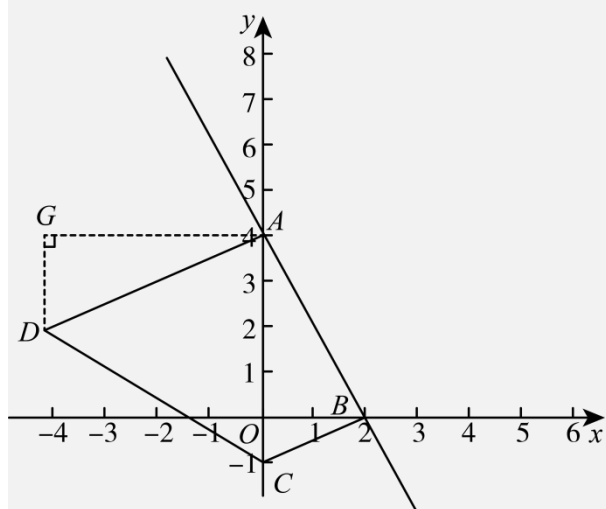
$$\because AD = AB,$$

$$\therefore k^2 + (5 + 2k)^2 = 20,$$

$$\text{解得: } k_1 = -2 - \sqrt{3}, k_2 = -2 + \sqrt{3},$$

$$\therefore D(-2 - \sqrt{3}, 3 + 2\sqrt{3}) \text{ 或 } D(-2 + \sqrt{3}, 3 - 2\sqrt{3});$$

当 $AD \parallel BC$ 时, 过点 A 作 $AG \perp y$ 轴, 过点 D 作 $DG \perp AG$, 如图所示:



$$\therefore \angle BAD = 90^\circ, \angle GAO = 90^\circ,$$

$$\therefore \angle DAO + \angle BAC = 90^\circ, \angle DAO + \angle GAD = 90^\circ,$$

$$\therefore \angle BAC = \angle GAD,$$

$$\because \angle G = \angle AOB = 90^\circ, AB = AD,$$

$$\therefore \triangle AGD \cong \triangle AOB (\text{AAS}),$$

$$\therefore AG = AO = 4, GD = BO = 2,$$

$$\therefore \text{点 } D \text{ 纵坐标为: } 4 - 2 = 2,$$

$$\therefore D(-4, 2);$$

综上所述可得: $D(-2 - \sqrt{3}, 3 + 2\sqrt{3})$ 或 $D(-2 + \sqrt{3}, 3 - 2\sqrt{3})$ 或 $D(-4, 2)$. $\cdots \cdots$ (3分)

26. (12分) 【详解】(1) 证明: $\because$ 直线 l 垂直平分 AB , 点 D 、 E 在直线 l 上,

$$\therefore DA = DB, EA = EB,$$

$\because$ 点 D 与点 E 关于点 O 对称,

$$\therefore OD = OE,$$

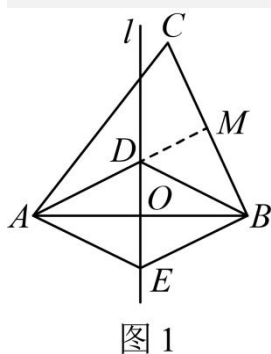
又 $AB \perp DE$, 即 AB 垂直平分 DE ,

$$\therefore AD = AE,$$

$$\therefore DA = DB = EA = EB,$$

$\therefore$ 四边形 $ADBE$ 是菱形; $\cdots\cdots$ (4 分)

(2) 解: 如图所示, 延长 AD 交 BC 于点 M ,



$$\because AB = AC = 2\sqrt{5}, BC = 4,$$

当 AD 平分 $\angle BAC$ 时,

$$\therefore AM \perp BC, \quad CM = MB = \frac{1}{2}BC = 2,$$

$$\text{在 Rt}\triangle ABM \text{ 中, } AM = \sqrt{AB^2 - BM^2} = \sqrt{20 - 4} = 4,$$

$$\text{设 } AD = x, \text{ 则 } BD = x, \quad DM = 4 - x,$$

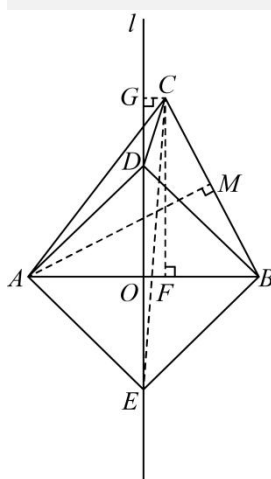
$$\text{在 Rt}\triangle DMB \text{ 中, } DB^2 = DM^2 + MB^2,$$

$$\therefore (4 - x)^2 + 2^2 = x^2,$$

$$\text{解得: } x = \frac{5}{2}, \text{ 即 } AD = \frac{5}{2},$$

$$\therefore \text{菱形 } ADBE \text{ 的周长为 } \frac{5}{2} \times 4 = 10, \cdots\cdots (4 \text{ 分})$$

(3) 解: 如图所示, 过点 C 分别作 l 和 AB 的垂线, 垂足分别为 G, F , 过点 A 作 $AM \perp BC$ 于点 M ,



又 $\because DE \perp AB$,

$\therefore$ 四边形 $GOFC$ 是矩形,

$$\therefore GO = CF, \quad CG = OF,$$

$\therefore$ 四边形 $ADBE$ 为正方形

$$\therefore DE = AB = 2\sqrt{5},$$

由 (2) 可得 $AM = 4$,

$$\therefore S_{\triangle ABC} = \frac{1}{2} BC \times AM = \frac{1}{2} AB \times CF,$$

$$\therefore CF = \frac{BC \times AM}{AB} = \frac{4 \times 4}{2\sqrt{5}} = \frac{8\sqrt{5}}{5},$$

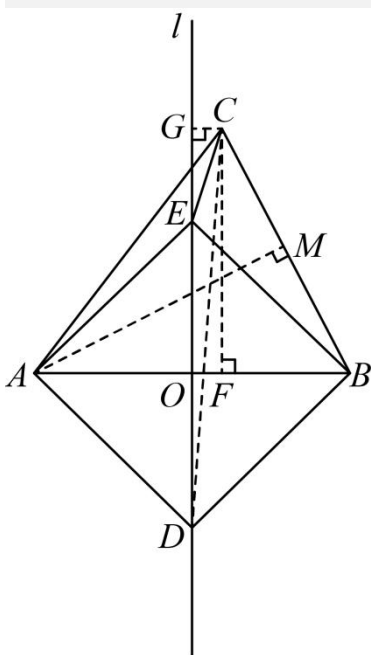
$$\therefore DG = OG - OD = \frac{8}{5}\sqrt{5} - \sqrt{5} = \frac{3}{5}\sqrt{5},$$

在 $\text{Rt}\triangle CFB$ 中, $FB = \sqrt{CB^2 - CF^2} = \sqrt{4^2 - \left(\frac{8\sqrt{5}}{5}\right)^2} = \frac{4\sqrt{5}}{5}$,

$$\therefore CG = OF = OB - FB = \sqrt{5} - \frac{4}{5}\sqrt{5} = \frac{\sqrt{5}}{5},$$

在 $\text{Rt}\triangle CDG$ 中, $CD = \sqrt{CG^2 + DG^2} = \sqrt{\left(\frac{\sqrt{5}}{5}\right)^2 + \left(\frac{3\sqrt{5}}{5}\right)^2} = \sqrt{2}$,

如图所示, 当 D 在 AB 的下方时,



同理可得: $GD = DE + GE = 2\sqrt{5} + \frac{3}{5}\sqrt{5} = \frac{13}{5}\sqrt{5}$,

$$CG = \frac{\sqrt{5}}{5},$$

$$\text{在 Rt}\triangle CGD \text{ 中, } CD = \sqrt{CG^2 + DG^2} = \sqrt{\left(\frac{\sqrt{5}}{5}\right)^2 + \left(\frac{13\sqrt{5}}{5}\right)^2} = \sqrt{34},$$

综上所述, $CD = \sqrt{2}$ 或 $\sqrt{34}$. …… (4 分)