

# 2025-2026 学年八年级数学上学期期中模拟卷

## 强化卷·参考答案

一、选择题：(本大题共 6 题，每题 4 分，共 24 分.下列各题四个选项中，有且只有一个选项是正确的，选择正确项的代号并填涂在答题卡的相应位置上.)

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| B | D | A | B | B | A |

二、填空题：(本大题共 12 题，每题 4 分，共 48 分.)

7.  $>$

8. 27

9.  $2\sqrt{3}$

10. -1

11.  $\sqrt{2}-1/-1+\sqrt{2}$

12. 9

13.  $-2x\sqrt{3y}$

14.  $x < 2$

15. 5

16.  $\frac{409}{990}$

17.  $\sqrt{5}-1$

18. 0

三、解答题：(本大题共 11 题，共 78 分.解答应写出文字说明，证明过程或演算步骤.)

19. (1) 解：无理数是无限不循环小数，不为 0，设无理数为  $a$ ，( $a$  不等于 0)，根据平方的性质， $a^2 > 0$ ，所以无理数的平方一定是正数，该说法正确.

(2) 解：例如  $\sqrt{2} \times \sqrt{2} = 2$ ， $\sqrt{2}$  是无理数，但它们的积 2 是有理数，所以“两个无理数的积仍为无理数”说法错误.

(3) 解：分数是有理数，有理数包括整数和分数，分数可以通过分子除以分母化为有限小数 (如  $\frac{1}{2} = 0.5$ )

或无限循环小数 (如  $\frac{1}{3}=0.\dot{3}$ ), 所以该说法正确.

$$20. (1) \text{ 解: } \sqrt{0.04} + \sqrt[3]{-8} + \sqrt{\frac{1}{4}} + |\sqrt{3}-2| + \sqrt{3}$$

$$= 0.2 + (-2) + \frac{1}{2} + (2 - \sqrt{3}) + \sqrt{3}$$

$$= 0.2 - 2 + 0.5 + 2 - \sqrt{3} + \sqrt{3}$$

$$= 0.7;$$

$$(2) \text{ 解: } \left(2\sqrt{12} - \sqrt{\frac{1}{3}}\right) \times \sqrt{6} - \frac{\sqrt{27} + \sqrt{12}}{\sqrt{3}}$$

$$= \left(4\sqrt{3} - \frac{\sqrt{3}}{3}\right) \times \sqrt{6} - \frac{3\sqrt{3} + 2\sqrt{3}}{\sqrt{3}}$$

$$= \frac{11\sqrt{3}}{3} \times \sqrt{6} - (3 + 2)$$

$$= \frac{11\sqrt{18}}{3} - 5$$

$$= \frac{33\sqrt{2}}{3} - 5$$

$$= 11\sqrt{2} - 5.$$

$$21. (1) \text{ 解: } 4\sqrt{5} + \sqrt{45} - \sqrt{8} + 4\sqrt{2}$$

$$= 4\sqrt{5} + 3\sqrt{5} - 2\sqrt{2} + 4\sqrt{2}$$

$$= 7\sqrt{5} + 2\sqrt{2};$$

$$(2) \text{ 解: } \frac{2}{\sqrt{3}+1} - \sqrt{6} \times \sqrt{2} + 1 \div (2 - \sqrt{3})$$

$$= \sqrt{3} - 1 - 2\sqrt{3} + \frac{1}{2 - \sqrt{3}}$$

$$= \sqrt{3} - 1 - 2\sqrt{3} + 2 + \sqrt{3}$$

$$= 1;$$

$$(3) \text{ 解: } 8x^2\sqrt{xy} \times 3\sqrt{\frac{y^2}{x}} \div 12\sqrt{\frac{x}{y}}$$

$$= 8x^2 \times 3 \div 12 \sqrt{xy \cdot \frac{y^2}{x} \div \frac{x}{y}}$$

$$= 2x \sqrt{x^2 \cdot xy \cdot \frac{y^2}{x} \cdot \frac{y}{x}}$$

$$= 2x\sqrt{xy^4}$$

$$= 2xy^2\sqrt{x};$$

$$\begin{aligned} (4) \text{ 解: } & \left( \frac{m-n}{\sqrt{m}+\sqrt{n}} + \frac{m-2\sqrt{mn}+n}{\sqrt{m}-\sqrt{n}} \right) \div \frac{1}{\sqrt{m}+\sqrt{n}} \\ &= \left[ \frac{m-n}{\sqrt{m}+\sqrt{n}} + \frac{(\sqrt{m}-\sqrt{n})^2}{\sqrt{m}-\sqrt{n}} \right] \cdot (\sqrt{m}+\sqrt{n}) \\ &= \frac{m-n}{\sqrt{m}+\sqrt{n}} \cdot (\sqrt{m}+\sqrt{n}) + (\sqrt{m}-\sqrt{n}) \cdot (\sqrt{m}+\sqrt{n}) \\ &= m-n+m-n \\ &= 2m-2n; \end{aligned}$$

$$(5) \text{ 解: } \sqrt{20}x - 4\sqrt{3} > 3\sqrt{5}x,$$

$$\text{化简得, } 2\sqrt{5}x - 4\sqrt{3} > 3\sqrt{5}x,$$

$$\text{移项得, } 2\sqrt{5}x - 3\sqrt{5}x > 4\sqrt{3},$$

$$\text{合并同类项得, } -\sqrt{5}x > 4\sqrt{3},$$

$$\text{系数化为 1 得, } x < -\frac{4\sqrt{3}}{\sqrt{5}}, \text{ 即 } x < -\frac{4\sqrt{15}}{5}.$$

$$22. \text{ 解: 原式} = \frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}{\sqrt{a}+\sqrt{b}} + \frac{(\sqrt{a}-2\sqrt{b})^2}{\sqrt{a}-2\sqrt{b}}$$

$$= \sqrt{a} - \sqrt{b} + \sqrt{a} - 2\sqrt{b}$$

$$= 2\sqrt{a} - 3\sqrt{b}$$

$$\text{当 } a = \frac{1}{2}, b = 2 \text{ 时,}$$

$$\text{原式} = 2 \times \frac{\sqrt{2}}{2} - 3\sqrt{2} = \sqrt{2} - 3\sqrt{2} = -2\sqrt{2}.$$

$$23. \text{ 解: } \left( \frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}} + \frac{1-x}{\sqrt{1-x^2}+x-1} \right) \left( \sqrt{\frac{1}{x^2}-1} - \frac{1}{x} \right)$$

$$= \left( \frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}} + \frac{1-x}{\sqrt{(1+x)(1-x)} + \sqrt{(1-x)^2}} \right) \frac{\sqrt{1-x^2}-1}{x}$$

$$= \left( \frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}} + \frac{\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} \right) \frac{\sqrt{1-x^2}-1}{x}$$

$$= \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} \frac{\sqrt{1-x^2}-1}{x}$$

$$\begin{aligned}
&= \frac{(\sqrt{1+x} + \sqrt{1-x})^2}{(\sqrt{1+x} - \sqrt{1-x})(\sqrt{1+x} + \sqrt{1-x})} \cdot \frac{\sqrt{1-x^2} - 1}{x} \\
&= \frac{(\sqrt{1+x} + \sqrt{1-x})^2}{(1+x) - (1-x)} \cdot \frac{\sqrt{1-x^2} - 1}{x} \\
&= \frac{(\sqrt{1+x} + \sqrt{1-x})^2}{2x} \cdot \frac{\sqrt{1-x^2} - 1}{x} \\
&= \frac{1+x + 2\sqrt{(1+x)(1-x)} + 1-x}{2x} \cdot \frac{\sqrt{1-x^2} - 1}{x} \\
&= \frac{2 + 2\sqrt{1-x^2}}{2x} \cdot \frac{\sqrt{1-x^2} - 1}{x} \\
&= \frac{1 + \sqrt{1-x^2}}{x} \cdot \frac{\sqrt{1-x^2} - 1}{x} \\
&= \frac{(1-x^2) - 1}{x^2} \\
&= -1
\end{aligned}$$

24. 解: (1) 设这个木箱的棱长为  $x\text{m}$ ,

则  $x^3 = 4$ ,

解得  $x = \sqrt[3]{4} \approx 1.6$ .

答: 这个木箱的棱长约  $1.6\text{m}$ .

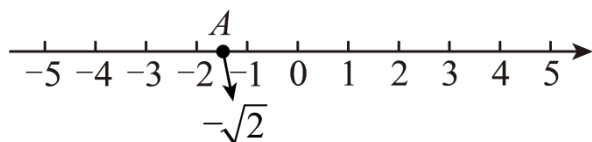
(2) 设这个篮球的半径为  $R\text{cm}$ ,

根据题意, 得  $\frac{4}{3}\pi R^3 = 9850$ ,

解得  $R \approx 13.3$ .

答: 这个篮球的半径约  $13.3\text{cm}$ .

25. (1) 解: 如图,



$\therefore$  点  $A$  即为所求;

(2) 解: 点  $B$  表示的数为  $-\sqrt{2} + 2$ , 其绝对值  $|\sqrt{2} + 2| = \sqrt{2} + 2$ ,

故答案为:  $-\sqrt{2} + 2$ ,  $-\sqrt{2} + 2$ ;

(3) 解: 根据数轴可知,  $-\sqrt{2} < \frac{1}{2}$ , 点  $C$  在点  $A$  的右侧,

故答案为:  $<$ , 右侧.

26. (1) 解: 由“美好数”的新定义可得,

则  $2+\sqrt{3}$  关于 1 的“美好数”是  $\frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$ ,

故答案为:  $2-\sqrt{3}$ ;

$$\begin{aligned} (2) \text{ 解: } & \frac{1}{\sqrt{9}+\sqrt{11}} + \frac{1}{\sqrt{11}+\sqrt{13}} + \cdots + \frac{1}{\sqrt{19}+\sqrt{21}} \\ &= \frac{1}{2}(-\sqrt{9}+\sqrt{11}-\sqrt{11}+\sqrt{13}+\cdots-\sqrt{19}+\sqrt{21}) \\ &= \frac{1}{2}(-\sqrt{9}+\sqrt{21}) \\ &= \frac{1}{2}(-3+11) \\ &= \frac{1}{2} \times 8 \\ &= 4; \end{aligned}$$

(3) 解:  $\sqrt{10}-1$  关于 9 的“美好数”  $y = \frac{9}{\sqrt{10}-1} = \sqrt{10}+1$ ,

$$\therefore 4y^2 - 8y + 2025$$

$$= 4(y-1)^2 + 2021$$

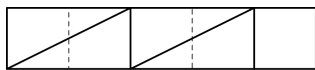
$$= 4(\sqrt{10}+1-1)^2 + 2021$$

$$= 4 \times (\sqrt{10})^2 + 2021$$

$$= 4 \times 10 + 2021$$

$$= 2061.$$

27. (1) 如图所示,

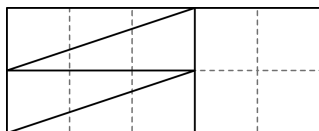


(2) ①  $\because$  长为 5, 宽为 2 的长方形的面积为  $5 \times 2 = 10$

$\therefore$  拼成的正方形的面积为 10

$\therefore$  该正方形的边长为  $\sqrt{10}$ ;

② 如图所示.



28. (1) 解: 设  $\sqrt{2} = \frac{a}{b}$ ,  $a$  与  $b$  是互素的两个整数, 且  $b \neq 0$ ,

$$\text{则 } 2 = \frac{a^2}{b^2}$$

$$\text{即 } a^2 = 2b^2.$$

因为  $b$  是整数且不为 0,

所以  $a$  是不为 0 的偶数.

设  $a = 2n$  ( $n$  是整数, 且  $n \neq 0$ ),

$$\text{则 } a^2 = 4n^2.$$

$$\text{所以 } b^2 = 2n^2.$$

所以  $b$  也是偶数, 与  $a, b$  是互素的整数矛盾.

所以  $\sqrt{2}$  是无理数.

(2) 设  $\sqrt[3]{6} = \frac{a}{b}$ ,  $a$  与  $b$  是互素的两个整数, 且  $b \neq 0$ , 则  $6 = \frac{a^3}{b^3}$ ,

$$\text{所以 } a^3 = 6b^3,$$

$\therefore a, b$  是整数且不为 0,

$\therefore a$  为 6 的倍数.

设  $a = 6n$  ( $n$  是整数),

$$\therefore a^3 = 216n^3,$$

$$\therefore b^3 = 36n^3,$$

$\therefore b$  也是 6 的倍数, 与  $a$  与  $b$  是互素的整数矛盾,

$\therefore \sqrt[3]{6}$  是无理数.

$$29. (1) \text{ 解: } \frac{3}{3-\sqrt{6}} = \frac{3(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6})} = \frac{3(3+\sqrt{6})}{9-6} = 3+\sqrt{6},$$

故答案为:  $3+\sqrt{6}$

$$\begin{aligned} (2) \text{ 解: } & \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{100}+\sqrt{99}} \\ &= \frac{1 \times (\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} + \frac{1 \times (\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} + \frac{1 \times (\sqrt{4}-\sqrt{3})}{(\sqrt{4}+\sqrt{3})(\sqrt{4}-\sqrt{3})} + \dots + \frac{1 \times (\sqrt{100}-\sqrt{99})}{(\sqrt{100}+\sqrt{99})(\sqrt{100}-\sqrt{99})} \end{aligned}$$

$$= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \dots + \frac{\sqrt{100}-\sqrt{99}}{100-99} = (\sqrt{2}-1) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + \dots + (\sqrt{100}-\sqrt{99})$$

$$= -1 + \sqrt{100}$$

$$= -1 + 10$$

$$= 9;$$

$$\begin{aligned} (3) \text{ 解: } & \frac{1}{3+\sqrt{3}} + \frac{1}{5\sqrt{3}+3\sqrt{5}} + \frac{1}{7\sqrt{5}+5\sqrt{7}} + \dots + \frac{1}{49\sqrt{47}+47\sqrt{49}} \\ &= \frac{1}{\sqrt{3}(\sqrt{3}+1)} + \frac{1}{\sqrt{3}\times 5(\sqrt{5}+\sqrt{3})} + \frac{1}{\sqrt{5}\times 7(\sqrt{7}+\sqrt{5})} + \dots + \frac{1}{\sqrt{47}\times 49(\sqrt{49}+\sqrt{47})} \\ &= \frac{\sqrt{3}-1}{\sqrt{3}(\sqrt{3}+1)(\sqrt{3}-1)} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}\times 5(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{5}\times 7(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} + \dots + \frac{\sqrt{49}-\sqrt{47}}{\sqrt{47}\times 49(\sqrt{49}+\sqrt{47})(\sqrt{49}-\sqrt{47})} \\ &= \frac{\sqrt{3}-1}{2\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{2\sqrt{3}\times 5} + \frac{\sqrt{7}-\sqrt{5}}{2\sqrt{5}\times 7} + \dots + \frac{\sqrt{49}-\sqrt{47}}{2\sqrt{47}\times 49} \\ &= \frac{1}{2} \times \left( 1 - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + \dots + \frac{1}{\sqrt{47}} - \frac{1}{\sqrt{49}} \right) \\ &= \frac{1}{2} \times \left( 1 - \frac{1}{\sqrt{49}} \right) \\ &= \frac{1}{2} \times \left( 1 - \frac{1}{7} \right) \\ &= \frac{3}{7}; \end{aligned}$$

$$\begin{aligned} (4) \text{ 解: } & \frac{\sqrt{5}-\sqrt{3}}{1+\sqrt{3}+\sqrt{5}+\sqrt{3}\times 5} + \frac{\sqrt{7}-\sqrt{5}}{1+\sqrt{5}+\sqrt{7}+\sqrt{5}\times 7} + \dots + \frac{\sqrt{2025}-\sqrt{2023}}{1+\sqrt{2023}+\sqrt{2025}+\sqrt{2023}\times 2025} \\ &= \frac{\sqrt{5}-\sqrt{3}}{1+\sqrt{3}+\sqrt{5}+\sqrt{3}\cdot\sqrt{5}} + \frac{\sqrt{7}-\sqrt{5}}{1+\sqrt{5}+\sqrt{7}+\sqrt{5}\cdot\sqrt{7}} + \dots + \frac{\sqrt{2025}-\sqrt{2023}}{1+\sqrt{2023}+\sqrt{2025}+\sqrt{2023}\cdot\sqrt{2025}} \\ &= \frac{(\sqrt{5}+1)-(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{5}+1)} + \frac{(\sqrt{7}+1)-(\sqrt{5}+1)}{(\sqrt{5}+1)(\sqrt{7}+1)} + \dots + \frac{(\sqrt{2025}+1)-(\sqrt{2023}+1)}{(\sqrt{2023}+1)(\sqrt{2025}+1)} \\ &= \frac{1}{\sqrt{3}+1} - \frac{1}{\sqrt{5}+1} + \frac{1}{\sqrt{5}+1} - \frac{1}{\sqrt{7}+1} + \dots + \frac{1}{\sqrt{2023}+1} - \frac{1}{\sqrt{2025}+1} \\ &= \frac{1}{\sqrt{3}+1} - \frac{1}{\sqrt{2025}+1} \end{aligned}$$

$$= \frac{\sqrt{3}-1}{(\sqrt{3}+1)(\sqrt{3}-1)} - \frac{1}{45+1}$$

$$= \frac{\sqrt{3}-1}{2} - \frac{1}{46}$$

$$= \frac{\sqrt{3}}{2} - \frac{12}{23}.$$