

2025-2026 学年八年级上学期期末模拟卷

数学 · 参考答案

一、选择题（本大题共 6 小题，每小题 3 分，满分 18 分。在每个小题给出的四个选项中，只有一项符合题目要求的）

1	2	3	4	5	6
B	A	C	A	B	D

二、填空题（本大题共 12 小题，每小题 2 分，满分 24 分）

7. 5230000 8. $x \geq -1$ 且 $x \neq \frac{1}{2}$ 9. $x < 2 - \sqrt{5}$ 10. $2\left(x - \frac{3 + \sqrt{17}}{4}y\right)\left(x - \frac{3 - \sqrt{17}}{4}y\right)$ 11. $<$

12. 102° 13. $12\sqrt{10}$ 14. $a < \frac{11}{6}$ 且 $a \neq \frac{1}{2}$ 15. 6 或 12 16. 36° 17. $2 + \frac{4\sqrt{3}}{3}$ 18. $\frac{8\sqrt{3}}{3} - 4$ 或 $\frac{8\sqrt{3}}{3} + 4$

三、解答题（本大题共 8 小题，58 分。解答应写出文字说明，证明过程或演算步骤）

19. (3 分)

【详解】移项 $3x(x-2) - (2-x) = 0$,

即 $3x(x-2) + (x-2) = 0$, …… (1 分)

$(x-2)(3x+1) = 0$ …… (2 分)

$\therefore x-2=0$ 或 $3x+1=0$,

解得 $x=2$ 或 $x=-\frac{1}{3}$. …… (3 分)

20. (6 分)

【详解】(1) 解: $\frac{1}{\sqrt{3}-2} - 4\sqrt{\frac{1}{2}} + (\sqrt{48} - \sqrt{24}) \div \sqrt{6}$

$= \frac{(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)} - 2\sqrt{2} + \sqrt{48} \div \sqrt{6} - \sqrt{24} \div \sqrt{6}$

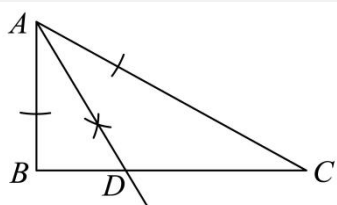
$= -\sqrt{3} - 2 - 2\sqrt{2} + 2\sqrt{2} - 2$

$= -\sqrt{3} - 4$; …… (3 分)

$$\begin{aligned}
 (2) \text{ 解: } & \left(\frac{a}{b} \sqrt{ab} \cdot \sqrt{a^3b} \right) \div \left(3 \sqrt{\frac{a}{b}} \right) (a > 0, b > 0) \\
 &= \left(\frac{a}{b} \sqrt{a^4b^2} \right) \div \left(3 \sqrt{\frac{a}{b}} \right) \\
 &= \left(\frac{a}{b} \sqrt{a^4b^2} \right) \times \left(\frac{1}{3} \sqrt{\frac{b}{a}} \right) \\
 &= \frac{a}{3b} \sqrt{a^3b^3} \\
 &= \frac{a}{3b} \cdot ab \sqrt{ab} \\
 &= \frac{1}{3} a^2 \sqrt{ab}. \quad \cdots \cdots (6 \text{ 分})
 \end{aligned}$$

21. (6 分)

【详解】(1) 解: 点 D 如图所示:



..... (2 分)

$$\begin{aligned}
 (2) \text{ 解: } & \because \angle ACB = 30^\circ, \angle B = 90^\circ, \\
 & \therefore \angle BAC = 180^\circ - 90^\circ - 30^\circ = 60^\circ, \\
 & \text{由 (1) 的作图痕迹, 得出 } AD \text{ 是 } \angle BAC \text{ 的角平分线,} \\
 & \therefore \angle BAD = \angle CAD = \frac{1}{2} \times 60^\circ = 30^\circ = \angle ACB, \\
 & \therefore AD = CD = 4 \text{ cm}, \\
 & \text{在 Rt} \triangle ABD \text{ 中, } BD = \frac{1}{2} AD = 2 \text{ cm}, \\
 & \therefore AB = \sqrt{AD^2 - BD^2} = 2\sqrt{3} (\text{cm}). \quad \cdots \cdots (6 \text{ 分})
 \end{aligned}$$

22. **新考向** (7 分)

$$\begin{aligned}
 \text{【详解】(1) 解: } & \because 3^4 = 81, (-3)^4 = 81, \\
 & \therefore 81 \text{ 的四次方根为 } \pm 3, \\
 & \because (-2)^5 = -32, \\
 & \therefore -32 \text{ 的五次方根为 } -2,
 \end{aligned}$$

故答案为: ± 3 ; -2 ; $\cdots$ (2 分)

(2) 解: 若 $\sqrt[3]{1-x}$ 有意义, 则 $1-x \geq 0$,

故 x 的取值范围是 $x \leq 1$;

若 $\sqrt[3]{1-x}$ 有意义, 则 x 的取值范围是任意实数,

故答案为: $x \leq 1$; 任意实数; $\cdots$ (4 分)

(3) 解: $\because \frac{1}{2}(2x-4)^4 - 8 = 0$,

$$\therefore \frac{1}{2}(2x-4)^4 = 8,$$

$$\therefore (2x-4)^4 = 16,$$

$$\therefore 2x-4 = 2 \text{ 或 } 2x-4 = -2,$$

$$\therefore x = 3 \text{ 或 } x = 1. \cdots (7 \text{ 分})$$

23.  (8 分)

【详解】(1) 解: 设与墙垂直的边 AD 的长为 x m, 则与墙平行的边 AB 的长为 $(170-2x)$ m,

$$\text{由题意可得: } x(170-2x) = 1500,$$

$$\text{解得: } x_1 = 10, x_2 = 75,$$

当 $x = 10$ 时, $170-2x = 170-2 \times 10 = 150 > 100$, 不符合题意;

当 $x = 75$ 时, $170-2x = 170-2 \times 75 = 20 < 100$, 符合题意;

$\therefore$ 与墙垂直的边 AD 的长为 75 m; $\cdots$ (4 分)

(2) 解: 不能, 理由如下:

设与墙垂直的边 AD 的长为 y m, 则与墙平行的边 AB 的长为 $(170-4y+3 \times 2)$ m,

$$\text{由题意可得 } y(170-4y+3 \times 2) = 2000,$$

$$\text{整理可得: } y^2 - 44y + 500 = 0,$$

$$\because \Delta = (-44)^2 - 4 \times 1 \times 500 = -64 < 0,$$

$\therefore$ 原方程没有实数根,

$\therefore$ 不能使长方形 $ABCD$ 面积扩大到 2000 平方米. $\cdots$ (8 分)

24. (8 分)

【详解】(1) 解: $\because x_1, x_2$ 是方程 $x^2 - 2(m+1)x + m^2 + 5 = 0$ 的两个实数根,

$$\therefore x_1 + x_2 = 2(m+1), \quad x_1 x_2 = m^2 + 5,$$

$\because x_1, x_2$ 是方程 $x^2 - 2(m+1)x + m^2 + 5 = 0$ 的两个实数根,

$$\therefore \Delta = b^2 - 4ac = [-2(m+1)]^2 - 4 \times (m^2 + 5) \times 1 \geq 0,$$

解得 $m \geq 2$.

$$\because (x_1 - 1)(x_2 - 1) = 28,$$

$$\therefore x_1 x_2 - x_1 - x_2 + 1 = 28,$$

$$\therefore m^2 + 5 - 2(m+1) - 27 = 0,$$

整理, 得 $m^2 - 2m - 24 = 0$,

解得 $m = 6$ 或 $m = -4$ (舍去),

故 m 的值为 6. $\cdots\cdots$ (3 分)

(2) 解: $\because x_1, x_2$ 是方程 $x^2 - 2(m+1)x + m^2 + 5 = 0$ 的两个实数根,

$$\therefore x_1 + x_2 = 2(m+1), \quad x_1 x_2 = m^2 + 5,$$

$\because$ 等腰三角形 ABC 的一边长为 7,

当 $x_1 = 7$ 时,

$$\therefore 7 + x_2 = 2(m+1), \quad 7x_2 = m^2 + 5,$$

$$\therefore 2(m+1) - 7 = \frac{m^2 + 5}{7},$$

整理, 得 $m^2 - 14m + 40 = 0$,

解得 $m_1 = 4, m_2 = 10$,

$$\text{当 } m = 4 \text{ 时, } x_2 = \frac{m^2 + 5}{7} = 3,$$

此时三角形的三边长为 7, 7, 3, 三角形存在,

故三角形的周长为 $7 + 7 + 3 = 17$;

$$\text{当 } m = 10 \text{ 时, } x_2 = \frac{m^2 + 5}{7} = 15,$$

此时三角形的三边长为 7, 7, 15, 三角形不存在;

同理可证, 当 $x_2 = 7$ 时, 三角形的周长为 17;

$\because x_1, x_2$ 是方程 $x^2 - 2(m+1)x + m^2 + 5 = 0$ 的两个实数根,

$$\therefore x_1 + x_2 = 2(m+1), \quad x_1 x_2 = m^2 + 5,$$

$\because$ 等腰三角形 ABC 的一边长为 7,

当 $x_1 = x_2$ 时,

$$\therefore \Delta = b^2 - 4ac = [-2(m+1)]^2 - 4 \times (m^2 + 5) \times 1 = 0,$$

解得 $m = 2$,

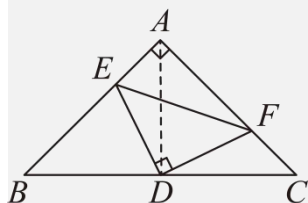
$$\therefore x_1 = x_2 = 3,$$

此时三角形的三边长为 7, 3, 3, 三角形不存在;

综上所述, 三角形周长为 17. $\cdots\cdots$ (8 分)

25. (8 分)

【详解】(1) 证明: 连接 AD ,



$\because \angle A = 90^\circ$, $AB = AC$, 点 D 是边 BC 的中点

$$\therefore AD = \frac{1}{2}BC = DC, \quad \angle C = \angle BAD = 45^\circ$$

又 $\because$ 在四边形 $AEDF$ 中,

$$\angle BAC = 90^\circ, \quad \angle EDF = 90^\circ$$

$$\therefore \angle AED + \angle AFD = 180^\circ$$

$$\text{又} \because \angle CFD + \angle AFD = 180^\circ$$

$$\therefore \angle AED = \angle CFD$$

在 $\triangle AED$ 和 $\triangle CFD$ 中,

$$\begin{cases} \angle C = \angle BAD \\ AD = DC \\ \angle AED = \angle CFD \end{cases}$$

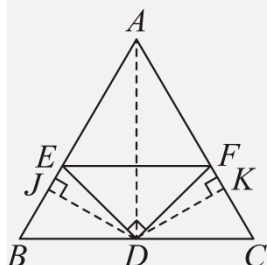
$$\therefore \triangle AED \cong \triangle CFD (AAS)$$

$$\therefore ED = FD$$

$\therefore \triangle EDF$ 为等腰直角三角形

$$\therefore \angle DEF = 45^\circ \cdots \cdots (4 \text{ 分})$$

(2)



连接 AD ，过点 D 作 $DJ \perp AB$ 于 J ，过点 D 作 $DK \perp AC$ 于 K

$\because$ 点 D 是边 BC 的中点， $\triangle ABC$ 是等边三角形，

$\therefore AD$ 平分 $\angle BAC$

$$\therefore DJ = KD$$

又 $\because$ 在等腰 $\text{Rt}\triangle EDF$ 中，

$$ED = FD$$

$\therefore$ 在 $\text{Rt}\triangle EDJ$ 和 $\text{Rt}\triangle FDK$ 中

$$\begin{cases} DJ = KD \\ ED = FD \end{cases}$$

$$\text{Rt}\triangle EDJ \cong \text{Rt}\triangle FDK (HL)$$

$$\therefore EJ = FK$$

同理可证 $\text{Rt}\triangle ADJ \cong \text{Rt}\triangle ADK (HL)$

$$\therefore AJ = AK$$

$$\therefore AJ - EJ = AK - FK$$

$$\therefore AE = AF \cdots \cdots (8 \text{ 分})$$

26. (12 分)

【详解】(1) 解：在 $\text{Rt}\triangle ABC$ 中 $\angle A = 30^\circ$ ， $\angle ACB = 90^\circ$ ，

$$\therefore AB = 2BC, \text{ 且 } AB^2 = AC^2 + BC^2,$$

$$\therefore (2BC)^2 = (2\sqrt{3})^2 + BC^2,$$

解得： $BC = 2$ ，负值舍去，

$$\therefore AB = 2BC = 4; \dots\dots (2 \text{ 分})$$

(2) 解: $\because BM \parallel AC$,

$$\therefore \angle BMD = \angle AED, \quad \angle MBD = \angle EAD,$$

∵ 点 D 为边 AB 的中点,

$$\therefore AD = BD,$$
$$\therefore \triangle BDM \cong \triangle ADE \text{ (AAS)},$$
$$\therefore MD = DE, \quad AE = BM = y,$$
$$\therefore DN \perp ME,$$

$\therefore DN$ 垂直平分 ME ,

$$\therefore MN = EN,$$
$$\because AC = 2\sqrt{3}, \quad CN = x,$$
$$\therefore CE = AC - AE = 2\sqrt{3} - y, \quad BN = BC + CN = 2 + x,$$

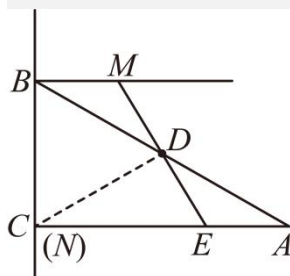
根据勾股定理得: $MN^2 = BM^2 + BN^2$, $EN^2 = CE^2 + CN^2$,

$$\therefore MN = EN,$$
$$\therefore BM^2 + BN^2 = CE^2 + CN^2,$$
$$\therefore y^2 + (2+x)^2 = (2\sqrt{3}-y)^2 + x^2,$$

整理得: $4x + 4\sqrt{3}y = 8$,

解得: $y = \frac{2\sqrt{3} - \sqrt{3}x}{3}$ (6 分)

(3) 解: 当 $AE = DE$ 时, 连接 CD , 如图所示:

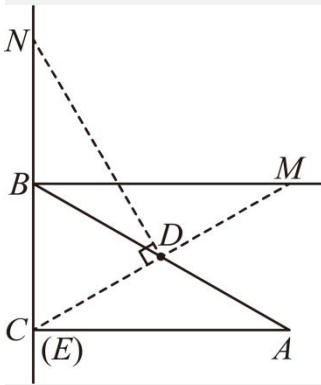


$\because AB = 4$, D 为 AB 的中点,

$$\therefore AD = BD = \frac{1}{2}AB = 2,$$
$$\therefore \angle ACD = \angle A = 30^\circ,$$
$$\therefore AE = DE,$$

$\therefore \angle ADE = \angle A = 30^\circ$,
 $\therefore \angle CED = \angle ADE + \angle A = 60^\circ$,
 $\therefore \angle CDE = 180^\circ - \angle DCE - \angle CED = 90^\circ$,
 $\therefore CD \perp ME$,
 $\therefore ND \perp ME$,
 $\therefore$ 此时点 N 与点 C 重合,
 $\therefore CN = 0$;

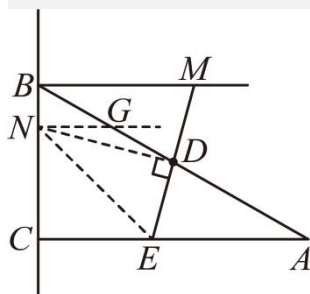
当 $DE = AD$ 时, 如图所示:



$\therefore$ Rt $\triangle ABC$ 中 $\angle A = 30^\circ$, $\angle ACB = 90^\circ$,
 $\therefore \angle ABC = 90^\circ - 30^\circ = 60^\circ$,
 $\therefore AD = DE$,
 $\therefore \angle AED = \angle A = 30^\circ$,
 $\therefore \angle BDE = \angle A + \angle AED = 60^\circ$,
 $\therefore AD = BD$,
 $\therefore DE = BD$,
 $\therefore \triangle DEB$ 为等边三角形,
 $\therefore \angle DBE = 60^\circ$,
 $\therefore \angle DBE = \angle DBC = 60^\circ$,
 $\therefore$ 此时点 E 与点 C 重合,
 $\therefore DN \perp ME$,
 $\therefore \angle NDE = 90^\circ$,
 $\therefore \angle END = 90^\circ - \angle BED = 30^\circ$,
 $\therefore EN = 2ED = 2CD = 2 \times 2 = 4$,

即此时 $CN = 4$;

当 $AE = AD = 2$ 时, 过点 N 作 $NG \parallel AC$ 交 AB 于点 G , 如图所示:



$$\therefore \angle BGN = \angle A = 30^\circ, \quad \angle BNG = \angle ACB = 90^\circ,$$

$$\therefore BN = \frac{1}{2} BG,$$

$$\because AD = AE,$$

$$\therefore \angle ADE = \angle AED = \frac{1}{2}(180^\circ - 30^\circ) = 75^\circ,$$

$$\because ND \perp ME,$$

$$\therefore \angle NDE = 90^\circ,$$

$$\therefore \angle BDN = 180^\circ - 90^\circ - 75^\circ = 15^\circ,$$

$$\therefore \angle DNG = \angle BGN - \angle BDN = 15^\circ,$$

$$\therefore \angle DNG = \angle BDN,$$

$$\therefore NG = GD,$$

$$\text{设 } BN = x, \text{ 则 } BG = 2x, \quad NG = GD = BD - BG = 2 - 2x,$$

$$\because BG^2 = NG^2 + BN^2,$$

$$\therefore (2x)^2 = (2 - 2x)^2 + x^2,$$

$$\text{解得: } x = 4 - 2\sqrt{3}, \quad x = 4 + 2\sqrt{3} \text{ (舍去),}$$

$$\therefore BN = 4 - 2\sqrt{3},$$

$$\therefore CN = 2 - BN = 2 - 4 + 2\sqrt{3} = 2\sqrt{3} - 2;$$

综上分析可得: CN 的长为 0 或 4 或 $2\sqrt{3} - 2$. …… (12 分)