

2025-2026 学年八年级上学期期中模拟卷 01

数学 · 参考答案

一、选择题 (本大题共 6 小题, 每小题 3 分, 满分 18 分。在每个小题给出的四个选项中, 只有一项符合题目要求的)

1	2	3	4	5	6
C	B	C	C	A	C

二、填空题 (本大题共 12 小题, 每小题 2 分, 满分 24 分)

7. 8 8. $-3a$ 9. $>$ 10. $4\sqrt{2}$
11. 1.026×10^{-3} 12. 0.7899 249.8 13. 千
14. 2 0 1 15. -0.112 16. $x > -2 - \sqrt{2}$
17. $\sqrt{6}$ 18. -1

三、解答题 (本大题共 8 小题, 58 分。解答应写出文字说明, 证明过程或演算步骤)

19. (4 分)

【详解】解: $\sqrt{75} - \frac{2}{3}\sqrt{27} - (\sqrt{3}-1)^0 + \frac{3}{\sqrt{3}+1}$

$$= 5\sqrt{3} - \frac{2}{3} \times 3\sqrt{3} - 1 + \frac{3(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} \dots\dots (1 \text{ 分})$$
$$= 5\sqrt{3} - 2\sqrt{3} - 1 + \frac{3\sqrt{3}-3}{2} \dots\dots (2 \text{ 分})$$
$$= 3\sqrt{3} - 1 + \frac{3\sqrt{3}}{2} - \frac{3}{2} \dots\dots (3 \text{ 分})$$
$$= \frac{9\sqrt{3}}{2} - \frac{5}{2} \dots\dots (4 \text{ 分})$$

20. (4 分)

【详解】解: $\left(\sqrt{24} - \sqrt{\frac{2}{3}}\right) \times \sqrt{3} - (3-2\sqrt{2})(3+2\sqrt{2})$

$$= \left(2\sqrt{6} - \frac{\sqrt{2}}{\sqrt{3}}\right) \times \sqrt{3} - [3^2 - (2\sqrt{2})^2] \dots\dots (1 \text{ 分})$$
$$= 2\sqrt{6} \times \sqrt{3} - \frac{\sqrt{2}}{\sqrt{3}} \times \sqrt{3} - (9-8) \dots\dots (2 \text{ 分})$$

$$= 6\sqrt{2} - \sqrt{2} - 1 \cdots \cdots (3 \text{ 分})$$

$$= 5\sqrt{2} - 1. \cdots \cdots (4 \text{ 分})$$

21. (4 分)

【详解】解： $\because$ 一个数的两个不同的平方根是 $3a+1$ 和 $a+11$,

$$\therefore 3a+1+a+11=0, \text{ 解得: } a=-3, \cdots \cdots (2 \text{ 分})$$

$$\therefore \text{这个数是 } (3a+1)^2 = [3 \times (-3)+1]^2 = 64,$$

$$\therefore \text{这个数的立方根是 } \sqrt[3]{64} = 4. \cdots \cdots (4 \text{ 分})$$

22. (6 分)

$$\text{【详解】解: 原式} = \frac{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}{\sqrt{a}+\sqrt{b}} + \frac{(\sqrt{a}-2\sqrt{b})^2}{\sqrt{a}-2\sqrt{b}} \cdots \cdots (1 \text{ 分})$$

$$= \sqrt{a} - \sqrt{b} + \sqrt{a} - 2\sqrt{b} \cdots \cdots (3 \text{ 分})$$

$$= 2\sqrt{a} - 3\sqrt{b} \cdots \cdots (4 \text{ 分})$$

$$\text{当 } a = \frac{1}{2}, b = 2 \text{ 时,}$$

$$\text{原式} = 2 \times \frac{\sqrt{2}}{2} - 3\sqrt{2} = \sqrt{2} - 3\sqrt{2} = -2\sqrt{2}. \cdots \cdots (6 \text{ 分})$$

23. (6 分)

$$\text{【详解】(1) 解: 根据前 4 个式子可得: } |\sqrt{35}-6| = |\sqrt{35}-\sqrt{36}| = \sqrt{36}-\sqrt{35} = 6-\sqrt{35},$$

这是第 35 个等式. $\cdots \cdots (2 \text{ 分})$

$$(2) \text{ 解: 由前 4 个等式可得第 } n \text{ 个等式为 } |\sqrt{n}-\sqrt{n+1}| = \sqrt{n+1}-\sqrt{n}. \cdots \cdots (4 \text{ 分})$$

$$(3) \text{ 解: } \because \frac{\sqrt{24}-1}{4} - 1 = \frac{\sqrt{24}-1}{4} - \frac{4}{4} = \frac{\sqrt{24}-5}{4} = \frac{\sqrt{24}-\sqrt{25}}{4} < 0,$$

$$\therefore \frac{\sqrt{24}-1}{4} < 1. \cdots \cdots (6 \text{ 分})$$

24. (11 分)

【详解】(1) 解: $2\sqrt{3}-5$ 与 $2\sqrt{3}+5$ 互为有理化因式,

$$\frac{3}{2\sqrt{5}} = \frac{3 \times \sqrt{5}}{2\sqrt{5} \times \sqrt{5}} = \frac{3\sqrt{5}}{10},$$

$$\text{故答案为: } 2\sqrt{3}+5; \frac{3\sqrt{5}}{10}. \cdots \cdots (3 \text{ 分})$$

$$\begin{aligned}
 (2) \text{ 解: } & \frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \cdots + \frac{1}{\sqrt{2025}+\sqrt{2023}} \\
 &= \frac{\sqrt{3}-1}{(\sqrt{3}+1)(\sqrt{3}-1)} + \frac{\sqrt{5}-\sqrt{3}}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + \frac{\sqrt{7}-\sqrt{5}}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} + \cdots + \frac{\sqrt{2025}-\sqrt{2023}}{(\sqrt{2025}+\sqrt{2023})(\sqrt{2025}-\sqrt{2023})} \\
 &= \frac{\sqrt{3}-1}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \cdots + \frac{\sqrt{2025}-\sqrt{2023}}{2} \\
 &= \frac{\sqrt{2025}-1}{2} \\
 &= 22; \cdots \cdots (7 \text{ 分})
 \end{aligned}$$

$$(3) \text{ 解: } \because \frac{a}{\sqrt{2}-1} - \frac{b}{\sqrt{2}} = a(\sqrt{2}+1) - \frac{\sqrt{2}}{2}b = a + \left(a - \frac{1}{2}b\right)\sqrt{2},$$

$$\text{且 } \frac{a}{\sqrt{2}-1} - \frac{b}{\sqrt{2}} = 2-3\sqrt{2},$$

$$\therefore a + \left(a - \frac{1}{2}b\right)\sqrt{2} = 2-3\sqrt{2},$$

$$\therefore \begin{cases} a=2 \\ a - \frac{1}{2}b = -3 \end{cases},$$

$$\text{解这个方程组, 得 } \begin{cases} a=2 \\ b=10 \end{cases},$$

$$\therefore a=2, b=10. \cdots \cdots (11 \text{ 分})$$

25. (11 分)

【详解】(1) 解: 根据题意可得:

$$\sqrt{13+2\sqrt{42}} = \sqrt{(\sqrt{6}+\sqrt{7})^2} = \sqrt{6} + \sqrt{7},$$

$$\sqrt{21+2\sqrt{110}} = \sqrt{(\sqrt{10}+\sqrt{11})^2} = \sqrt{10} + \sqrt{11},$$

故答案为: $\sqrt{6}+\sqrt{7}, \sqrt{10}+\sqrt{11}. \cdots \cdots (4 \text{ 分})$

$$(2) \text{ 解: } \sqrt{(2n-1)+2\sqrt{n^2-n}} = \sqrt{(\sqrt{n-1}+\sqrt{n})^2} = \sqrt{n-1} + \sqrt{n},$$

故答案为: $\sqrt{n-1}+\sqrt{n}. \cdots \cdots (6 \text{ 分})$

(3) 解: 根据题意可得:

$$\frac{1}{\sqrt{3+2\sqrt{2}}} + \frac{1}{\sqrt{5+2\sqrt{6}}} + \frac{1}{\sqrt{7+2\sqrt{12}}} + \cdots + \frac{1}{\sqrt{(2n-1)+2\sqrt{n^2-n}}}$$

$$= \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{n-1}+\sqrt{n}}$$

$$\begin{aligned}
&= -(\sqrt{1}-\sqrt{2}) - (\sqrt{2}-\sqrt{3}) - (\sqrt{3}-\sqrt{4}) - \cdots - (\sqrt{n-1}-\sqrt{n}) \\
&= -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + \sqrt{4} - \cdots - \sqrt{n-1} + \sqrt{n} \\
&= -1 + \sqrt{n},
\end{aligned}$$

故答案为: $-1 + \sqrt{n}$. …… (11 分)

26. **新考向** (12 分)

【详解】(1) 解: 由题意, 设 $a = x$, $b = \frac{9}{x}$,

$$\therefore \text{由 } a+b \geq 2\sqrt{ab}, \text{ 得 } x + \frac{9}{x} \geq 2\sqrt{x \cdot \frac{9}{x}} = 6,$$

$\therefore$ 当且仅当 $x = \frac{9}{x}$, 即 $x = 3$ (负值舍去) 时, 代数式取到最小值, 最小值为 6.

故答案为: 6; 3; …… (4 分)

(2) 解: 由题意, 设花圃的宽为 x 米, 则长为 $\frac{108}{x}$ 米,

$$\therefore \text{所用的篱笆} = 3x + \frac{108}{x},$$

$$\text{又令 } a = 3x, \quad b = \frac{108}{x},$$

$$\therefore \text{由 } a+b \geq 2\sqrt{ab},$$

$$\therefore 3x + \frac{108}{x} \geq 2\sqrt{3x \cdot \frac{108}{x}} = 36.$$

$\therefore$ 当且仅当 $3x = \frac{108}{x}$, 即 $x = 6$ (负值舍去) 时, 代数式取到最小值, 最小值为 36,

答: 所用的篱笆至少为 36 米. …… (8 分)

(3) 解: 由题意, 设 $S_{\triangle COB} = m$,

$\therefore \triangle COB$ 与 $\triangle COD$ 底边上的高相等, $\triangle AOB$ 与 $\triangle AOD$ 底边上的高相等,

$$\therefore \frac{OB}{OD} = \frac{S_{\triangle COB}}{S_{\triangle COD}} = \frac{S_{\triangle AOB}}{S_{\triangle AOD}},$$

$$\therefore \frac{m}{4} = \frac{9}{S_{\triangle AOD}}.$$

$$\therefore S_{\triangle AOD} = \frac{36}{m}.$$

$$\therefore S_{\text{四边形}ABCD} = 9 + 4 + m + \frac{36}{m} = 13 + m + \frac{36}{m}.$$

$$\text{又} \therefore m + \frac{36}{m} \geq 2\sqrt{m \times \frac{36}{m}} = 12,$$

$$\therefore S_{\text{四边形}ABCD} \geq 13 + 12 = 25,$$

$\therefore$ 当 $m = \frac{36}{m}$ 时, 即 $m = 6$ (负值舍去) 时取等号.

$\therefore$ 四边形 $ABCD$ 面积的最小值为 25. $\cdots\cdots$ (12 分)