

黄浦区 2025 年九年级学业水平考试模拟考

参考答案与评分建议

一、选择题: (本大题 6 小题, 每小题 4 分, 满分 24 分)

1.D; 2.B; 3.D; 4.C; 5.C; 6.A.

二、填空题: (本大题共 12 题, 每题 4 分, 满分 48 分)

7. 8; 8. $x^2 - 4y^2$; 9. $\frac{1}{2}$; 10. $m \geq -\frac{9}{4}$; 11. 8; 12. $\frac{2}{3}\vec{b} - \frac{2}{3}\vec{a}$;

13. 3π ; 14. $y = 8x + 2$; 15. 5; 16. 3; 17. $y = \frac{a}{k}x^2 + bx + kc (a \neq 0)$;

18. $\frac{15 \pm 2\sqrt{5}}{4}$.

三、解答题: (本大题共 7 题, 满分 78 分)

19. 解: 原式 $= \sqrt{2} - 1 + 3\sqrt{3} - (2 - \sqrt{3}) + 1$, (8 分)

$= \sqrt{2} + 4\sqrt{3} - 2$ (2 分)

20. 解: $2(x-3) + 6 = x + 3 + x^2 - 9$, (4 分)

整理得, $x^2 - x - 6 = 0$, (3 分)

解得 $x_1 = 3, x_2 = -2$ (2 分)

经检验 $x_1 = 3$ 是原方程的增根, 所以原方程的解是 $x = -2$ (1 分)

21. 解: (1) $\because$ 双曲线 $y = \frac{8}{x} (x > 0)$ 经过点 $P(2, n)$, $\therefore 2n = 8, n = 4, \therefore P(2, 4)$. (2 分)

$\because$ 点 P 在直线 l 上, $\therefore 2k + 3 = 4, \therefore k = \frac{1}{2}, \therefore y = \frac{1}{2}x + 3$ (2 分)

(2) $\because \angle BOC = \angle ABO, \therefore AB \parallel CO$ (2 分)

$\therefore$ 直线 CO 的表达式为 $y = \frac{1}{2}x$ (2 分)

$\therefore \begin{cases} y = \frac{1}{2}x, \\ y = \frac{8}{x}. \end{cases} \therefore x = 4$ (负根舍), $\therefore \begin{cases} x = 4, \\ y = 2 \end{cases} \therefore C(4, 2)$ (2 分)

22. (1) 2.6; (2) $2.6A + 2.4B + 3.5C + 1.5D$; (3 分, 3 分)

(3) 3631.4, 4340.8; (1 分, 1 分)

能提出合理的优化“阅读之星”评选方案的建议 (2 分)

23. 证明: (1) 方法 1: $\because AD = CD, AE = CE, \therefore EM \perp AC, AM = CM$ (3 分)

$\because \angle BAC = 90^\circ, \therefore \angle BAC = \angle CMN, \therefore AB \parallel MN$ (1 分)

$\therefore \frac{CM}{AM} = \frac{CN}{BN}, \therefore BN = CN$ (2 分)

方法 2: $\because AD = CD, AE = CE, ED = ED, \therefore \triangle AED \cong \triangle CED, \therefore \angle AED = \angle CED$ (2 分)

又 $AE = CE, \therefore EM \perp AC, AM = CM$ (1 分)

$\because \angle BAC = 90^\circ$, $\therefore \angle BAC = \angle CMN$, $\therefore AB \parallel MN$. (1 分)

$\therefore \frac{CM}{AM} = \frac{CN}{BN}$, $\therefore BN = CN$. (2 分)

(2) $\because BC^2 = 2AB \cdot NE$, 又 $CN = \frac{1}{2}BC$, $\therefore \frac{BC}{AB} = \frac{NE}{CN}$. (2 分)

$\because AB \parallel MN$, $\therefore \angle ABC = \angle MNC$, $\therefore \triangle ABC \sim \triangle CNE$. (2 分)

$\therefore \angle ACB = \angle NEC$. (2 分)

24. 解: (1) $\because A(2,0)$, $AB=6$, $\therefore B(-4,0)$. (1 分)

$\therefore \begin{cases} -1+2b+c=0, \\ -4-4b+c=0 \end{cases}$, 解得 $\begin{cases} b=-\frac{1}{2}, \\ c=2 \end{cases}$. (2 分)

抛物线表达式 $y = -\frac{1}{4}x^2 - \frac{1}{2}x + 2$. (1 分)

(2) $\because$ 抛物线经过点 $A(2,0)$, $\therefore c = 1 - 2b$. (1 分)

$x_p = -\frac{b}{2 \times \left(-\frac{1}{4}\right)} = 2b$, $y_p = \frac{4 \times \left(-\frac{1}{4}\right)c - b^2}{4 \times \left(-\frac{1}{4}\right)} = b^2 + c = b^2 - 2b + 1$. (2 分)

$\therefore P(2b, b^2 - 2b + 1)$. (1 分)

(3) 方法 1: $\because A(2,0)$, $x_p = 2b$, $\therefore B(4b-2,0)$. (1 分)

$\because BC=EC$, $CO \perp BE$, $\therefore EO=BO$, $\therefore E(2-4b,0)$.

$\because C(0,1-2b)$, $\therefore OC=1-2b$, 又 $OE=2-4b$, $\therefore \tan \angle CEO = \frac{1}{2}$. (1 分)

过点 P 作 $PH \perp x$ 轴于点 H , $\therefore \frac{PH}{EH} = \frac{b^2 - 2b + 1}{2 - 4b - 2b} = \frac{1}{2}$, $\therefore b_1 = 0$ (舍), $b_2 = -1$. (1 分)

$\therefore c = 3$, 抛物线的表达式为 $y = -\frac{1}{4}x^2 - x + 3$. (1 分)

方法 2: $\because A(2,0)$, $x_p = 2b$, $\therefore B(4b-2,0)$. (1 分)

$\because BC=EC$, $CO \perp BE$, $\therefore BO=EO$, $\therefore E(2-4b,0)$.

$\therefore$ 直线 EC 的表达式为 $y = -\frac{1}{2}x + 1 - 2b$. (1 分)

又点 P 在直线 EC 上, $\therefore -\frac{1}{2} \cdot 2b + 1 - 2b = b^2 - 2b + 1$, $\therefore b_1 = 0$ (舍), $b_2 = -1$. (1 分)

$\therefore c = 3$, 抛物线的表达式为 $y = -\frac{1}{4}x^2 - x + 3$. (1 分)

25. 解: (1) $\odot O$ 与边 AB 相切.

过点 O 作 $OG \perp AB$, 垂足为点 G . (1 分)

$$\because AC=5, \frac{CO}{OD}=\frac{4}{5}, \therefore OC=\frac{20}{9}, OA=\frac{25}{9}. \quad (1 \text{ 分})$$

$$\because \cos A=\frac{3}{5}, \therefore \sin A=\frac{4}{5}, \therefore OG=OA \cdot \sin A=\frac{20}{9}. \quad (1 \text{ 分})$$

$\therefore OG=OC$, $\therefore \odot O$ 与边 AB 相切. (1 分)

(2) 方法 1: 过点 C 作 $CM \perp AB$, 垂足为点 M .

$$\because AC=5, \cos A=\frac{3}{5}, \therefore AM=3, CM=4. \because AB=7, \therefore BM=4, \therefore \angle B=45^\circ, BC=4\sqrt{2}. \quad (1 \text{ 分})$$

$$\because \widehat{CE}=\widehat{CF}, CD \text{ 过圆心 } O, \therefore CO \perp EF. \quad (1 \text{ 分})$$

$$\because CO=EO, \therefore \angle CEH=45^\circ, \therefore \angle CEH=\angle B, \text{ 又 } \angle ECH=\angle ECH, \therefore \triangle CEH \sim \triangle CBA. \quad (1 \text{ 分})$$

$$\therefore \frac{EH}{AB}=\frac{CE}{BC}, \therefore EH=\frac{7CE}{4\sqrt{2}}=\frac{7}{4}EO, \therefore EF=2EO, \therefore \frac{EH}{EF}=\frac{7}{8}. \quad (1 \text{ 分})$$

方法 2: 过点 C 作 $CM \perp AB$, 垂足为点 M .

$$\because AC=5, \cos A=\frac{3}{5}, \therefore AM=3, CM=4. \because AB=7, \therefore BM=4, \therefore \angle MCB=45^\circ. \quad (1 \text{ 分})$$

$$\because \widehat{CE}=\widehat{CF}, CD \text{ 过圆心 } O, \therefore CO \perp EF. \quad (1 \text{ 分})$$

$$\because CO=EO, \therefore \angle ECO=45^\circ, \therefore \angle ACM=\angle OCH, \text{ 又 } \angle COH=\angle AMC, \therefore \triangle COH \sim \triangle CMA. \quad (1 \text{ 分})$$

$$\therefore \frac{OH}{AM}=\frac{OC}{MC}, \therefore OH=\frac{3}{4}OC, \therefore EH=\frac{7}{4}OC, \therefore EF=2OC, \therefore \frac{EH}{EF}=\frac{7}{8}. \quad (1 \text{ 分})$$

(3) 设 $BD=x$.

当 $\odot O$ 与线段 DP 相切时, 切点记为点 N , 联结 ON .

$$\therefore ON \perp DP, ON=CO, \therefore \frac{CO}{OD}=\frac{4}{5}, \therefore \sin \angle ODN=\frac{ON}{OD}=\frac{4}{5}. \quad (1 \text{ 分})$$

$$\because DP \parallel AC, \therefore \angle ACD=\angle ODN, \therefore \sin \angle ACD=\frac{4}{5},$$

$$\text{又 } \sin \angle A=\frac{4}{5}, \therefore \angle ACD=\angle A, \therefore CD=AD. \quad (1 \text{ 分})$$

$$\because CM \perp AB, CM=4, \therefore CD=\sqrt{4^2+(4-x)^2}, \text{ 又 } AD=7-x,$$

$$\therefore \sqrt{4^2+(4-x)^2}=7-x, \therefore x=\frac{17}{6}. \quad (1 \text{ 分})$$

当点 P 在 $\odot O$ 上时, 分别过点 O 、 D 作 $OQ \perp CB$, $DR \perp BC$, 垂足为点 Q 、 R .

$$\therefore CQ=PQ, \because DP \parallel AC, \therefore \frac{CP}{BC}=\frac{AD}{AB}, \therefore CP=\frac{4\sqrt{2}(7-x)}{7}, \therefore CQ=2\sqrt{2}-\frac{2\sqrt{2}}{7}x. \quad (1 \text{ 分})$$

$$\because DR \perp BC, \angle B=45^\circ, \therefore BR=\frac{\sqrt{2}}{2}x, \therefore CR=4\sqrt{2}-\frac{\sqrt{2}}{2}x.$$

$$\because \frac{CO}{OD}=\frac{4}{5}, \text{ 又 } OQ \parallel DR, \therefore \frac{CO}{CD}=\frac{CQ}{CR}=\frac{4}{9} \therefore \frac{2\sqrt{2}-\frac{2\sqrt{2}}{7}x}{4\sqrt{2}-\frac{\sqrt{2}}{2}x}=\frac{4}{9}, \therefore x=\frac{7}{2}. \quad (1 \text{ 分})$$

$\therefore$ 当 $\frac{7}{2} < BD < 7$ 时点 P 在 $\odot O$ 内, $\odot O$ 与线段 DP 只有一个交点.

综上所述, 当 $BD = \frac{17}{6}$ 或 $\frac{7}{2} < BD < 7$ 时, $\odot O$ 与线段 DP 只有一个交点. (1 分)