

青浦区 2024 学年第二学期学业质量调研

九年级数学试卷参考答案

Q2025.4

一、选择题: (本大题共 6 题, 每小题 4 分, 满分 24 分)

1. C; 2. C; 3. B; 4. D; 5. A; 6. D.

二、填空题: (本大题共 12 题, 每小题 4 分, 满分 48 分)

7. $9a^2$; 8. $2(m+4)(m-4)$; 9. $x \neq 1$; 10. $x=6$; 11. $x>6$; 12. 7.675×10^5 ;13. $90(1+x)^2=120$; 14. 320; 15. $\frac{1}{3}\vec{b}-\vec{a}$; 16. $\sqrt{3}$; 17. $\frac{6\sqrt{5}}{5}$ 或 $\frac{12\sqrt{5}}{5}$;18. $t \leq -\frac{5}{2}$ 或 $-\frac{1}{2} \leq t < 0$.

三、解答题:

19. 解: 原式 $= 4 - (2 - \sqrt{3}) + \sqrt{3} + 1 - 3\sqrt{3}$ (8 分)
 $= 3 - \sqrt{3}$ (2 分)20. 解: 由②得, $x = y - 1$. ③ (2 分)把 $x = y - 1$ 代入①, 得 $(y - 1)^2 + 2y - 2 = 0$.整理, 得 $y^2 = 1$ (2 分)解这个方程, 得 $y_1 = 1$, $y_2 = -1$ (2 分)将 $y_1 = 1$ 代入③, 得 $x_1 = 0$;将 $y_2 = -1$ 代入③, 得 $x_2 = -2$ (2 分)所以, 原方程组的解是 $\begin{cases} x_1 = 0, \\ y_1 = 1; \end{cases} \begin{cases} x_2 = -2, \\ y_2 = -1. \end{cases}$ (2 分)21. 解: (1) 将点 $A(-2, 4)$ 代入反比例函数 $y = \frac{k_2}{x}$, 得 $k_2 = -8$, $y = -\frac{8}{x}$ (1 分)将点 $B(4, n)$ 代入反比例 $y = -\frac{8}{x}$, 得 $n = -2$, $B(4, -2)$ (1 分)将点 $A(-2, 4)$ 、 $B(4, n)$ 分别代入一次函数 $y = k_1x + b$, 得 $\begin{cases} -2k_1 + b = 4, \\ 4k_1 + b = -2. \end{cases}$ 解这个方程组, 得 $\begin{cases} k_1 = -1, \\ b = 2. \end{cases}$ (2 分)一次函数解析式为 $y = -x + 2$ (1 分)(2) 一次函数 $y = -x + 2$ 的图像与 x 轴交点 $C(2, 0)$ (1 分) $S_{\triangle AOB} = S_{\triangle AOC} + S_{\triangle BOC} = \frac{1}{2} \times 2 \times 4 + \frac{1}{2} \times 2 \times 2 = 6$ (4 分)

22. 解: (1) 略.

(2) 设边 FG 的中点为 H . $\because$ 点 O 为重心.

$$\therefore OH = \frac{1}{3}EH = \frac{5}{3}. \quad \dots\dots\dots (1 \text{ 分})$$

$$\text{在 } Rt\triangle FOH \text{ 中, } \because FH = 1, \quad OH = \frac{5}{3}, \quad \therefore OF = \frac{\sqrt{34}}{3}. \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore \sin \angle OFG = \frac{OH}{OF} = \frac{5\sqrt{34}}{34}. \quad \dots\dots\dots (2 \text{ 分})$$

$$23. \text{ 证明: (1) } \because AD \parallel BC, \quad \therefore \frac{OA}{OE} = \frac{OD}{OB}. \quad \dots\dots\dots (1 \text{ 分})$$

$$\because OA \cdot BE = OE \cdot CE, \quad \therefore \frac{OA}{OE} = \frac{CE}{BE}. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{OD}{OB} = \frac{CE}{BE}. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore OE \parallel CD \text{ 即 } AE \parallel CD.$$

$$\because AD \parallel BC, \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore \text{四边形 AECD 是平行四边形.} \quad \dots\dots\dots (1 \text{ 分})$$

$$(2) \because \angle BDE = \angle DAE, \quad \angle OED = \angle DEA,$$

$$\therefore \triangle DOE \sim \triangle ADE. \quad \therefore \frac{DE}{AE} = \frac{OD}{AD}. \quad \dots\dots\dots (2 \text{ 分})$$

$$\because AD \parallel BC. \quad \therefore \frac{OD}{OB} = \frac{AD}{BE}, \quad \text{即 } \frac{OD}{AD} = \frac{OB}{BE}. \quad \therefore \frac{DE}{AE} = \frac{OB}{BE}. \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore \text{四边形 AECD 是平行四边形.} \quad \therefore AE = CD. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{DE}{CD} = \frac{OB}{BE}. \quad \text{即 } OB \cdot CD = BE \cdot DE. \quad \dots\dots\dots (1 \text{ 分})$$

$$24. \text{ 解: (1) 由直线 } y = x + 4 \text{ 与 } x \text{ 轴、} y \text{ 轴分别交于点 } A、B, \text{ 得 } A(-4, 0), \quad B(0, 4). \quad (1 \text{ 分})$$

$$\text{将点 } A(-4, 0)、B(0, 4) \text{ 代入 } L_1, \text{ 得 } \begin{cases} 16a - 4b + c = 0, \\ c = 4, \\ -\frac{b}{2a} = -3. \end{cases} \quad \text{解得: } \begin{cases} a = \frac{1}{2}, \\ b = 3, \\ c = 4. \end{cases} \quad \dots\dots\dots (2 \text{ 分})$$

$$\text{所以, } L_1 \text{ 的表达式为 } y = \frac{1}{2}x^2 + 3x + 4. \quad \dots\dots\dots (1 \text{ 分})$$

$$(2) \because \text{抛物线 } L_1 \text{ 的对称轴是直线 } x = -3,$$

$$\text{当 } x = -3 \text{ 时, } y = \frac{1}{2} \times (-3)^2 + 3 \times (-3) + 4 = -\frac{1}{2},$$

$$\therefore \text{顶点 } P \text{ 的坐标是 } (-3, -\frac{1}{2}). \quad \dots\dots\dots (1 \text{ 分})$$

在对称轴 $x = -3$ 上取点 $E(-3, 3)$, 对称轴与 x 轴的交点记为点 H .

在 $Rt\triangle EOH$ 中, $\because OH = EH = 3$, $\therefore \angle EOH = \angle HEO = 45^\circ$.

$\because \angle POQ = 45^\circ$, $\therefore \angle EOH = \angle POQ$.

$\because$ 点 P 在 x 轴的下方, $\therefore$ 点 Q 在线段 EH 上. $\therefore \angle EOQ = \angle POH$ (1 分)

过点 Q 作 $QG \perp OE$, 垂足为点 G , 则 $QG = EG$.

$\because \tan \angle POH = \frac{HP}{OH} = \frac{1}{6}$, $\therefore \tan \angle OQE = \frac{1}{6}$ (1 分)

设 $QG = EG = x$, 则 $OG = 6x$, $\because EG + OG = OE$. $\therefore x + 6x = 3\sqrt{2}$.

$\therefore x = \frac{3}{7}\sqrt{2}$. 即 $EG = \frac{3}{7}\sqrt{2}$. $\therefore QE = \frac{6}{7}$. $\therefore QH = \frac{15}{7}$, 点 Q 的坐标为 $(-3, \frac{15}{7})$ (1 分)

(3) $\because PN \parallel AB$, $PM \parallel AO$,

$\therefore$ 点 M 在射线 BA 上, 且点 M 的纵坐标与点 P 相同 $y = -\frac{1}{2}$.

$\therefore$ 将 $y = -\frac{1}{2}$ 代入直线 $y = x + 4$, 得 $x = -\frac{9}{2}$,

即, 点 A 向左平移 $\frac{1}{2}$ 个单位, 再向下平移 $\frac{1}{2}$ 个单位得到点 M (1 分)

$\therefore$ 点 P 向左平移 $\frac{1}{2}$ 个单位, 再向下平移 $\frac{1}{2}$ 得到点 N (1 分)

$\therefore$ 点 N 的坐标为 $(-\frac{7}{2}, -1)$ (1 分)

$\therefore$ 平移后的抛物线解析式为 $y = \frac{1}{2}(x + \frac{7}{2})^2 - 1$ (1 分)

25. 解: (1) $\because OB = OC$, $\angle ABC = 60^\circ$

$\therefore \triangle BOC$ 是等边三角形, $OC = BC$ (1 分)

$\because OD \parallel BC$,

$\therefore \angle DOB = \angle ABC = 60^\circ$ (1 分)

同理, $\triangle BOD$ 是等边三角形, $OD = DB$ (1 分)

又 $\because OD = OC$

$\therefore OC = BC = OD = DB$. $\therefore$ 四边形 $OCBD$ 是菱形. (1 分)

(2) ① $\because OD = OC$, $\therefore \angle ODC = \angle OCD$.

$\because OD \parallel BC$, $\therefore \angle ODC = \angle DCB$.

$\therefore \angle OCB = 2\angle OCD$ (1 分)

$\because OC = OB$, $\therefore \angle OBC = \angle OCB = 2\angle OCD$.

$\therefore \angle CAB = 2\angle OCD$, $\therefore \angle CAB = \angle CBA$. $\therefore CA = CB$.

又 $\because OA = OB$, $\therefore \angle BOC = 90^\circ$ (1 分)

$\because DE \perp OB$, $\therefore \angle OED = 90^\circ$. $\therefore \angle OED = \angle BOC$.

$\therefore OC \parallel DE$. $\therefore \frac{EF}{OF} = \frac{DE}{OC}$ (1 分)

$\because OB = OC$, $\angle BOC = 90^\circ$, $\therefore \angle OBC = 45^\circ$.

$\therefore \angle DOE = \angle OBC = 45^\circ$.

$\therefore DE = \frac{\sqrt{2}}{2} OD = \frac{5\sqrt{2}}{4}$, (1 分)

$\therefore \frac{EF}{OF} = \frac{DE}{OC} = \frac{\sqrt{2}}{2}$

$\therefore EF = \frac{10 - 5\sqrt{2}}{4}$ (1 分)

(2) ②过点 O 作 $OH \perp BC$ 于点 H , 得 $BH = CH$, 则 $\triangle ODE \cong \triangle OBH$ (1 分)

$\therefore S_{\triangle ODE} = S_{\triangle OBH} = \frac{1}{2} S_{\triangle BOC} = \frac{1}{4} S_{\triangle ABC}$,

$\therefore S_{\triangle ABC} = 4S_1$ (1 分)

$\therefore S_{\text{四边形}ACBD} = S_{\triangle ABC} + S_{\triangle ABD} = 7S_1$, $S_{\triangle BOD} = S_{\triangle AOD} = \frac{1}{2} S_{\triangle ABD}$,

$\therefore S_{\triangle BOD} = \frac{3}{2} S_1$ (1 分)

$\therefore \frac{S_{\triangle BOD}}{S_{\triangle DOE}} = \frac{OB}{OE} = \frac{3}{2}$, $OE = \frac{5}{3}$.

$\therefore DE = \sqrt{OD^2 - OE^2} = \sqrt{\left(\frac{5}{2}\right)^2 - \left(\frac{5}{3}\right)^2} = \frac{5}{6}\sqrt{5}$.

$\therefore \triangle ODE \cong \triangle OBH$,

$\therefore OH = DE = \frac{5}{6}\sqrt{5}$ (1 分)

$\therefore$ 在 $\triangle ABC$ 中, $OB = OA$, $BH = CH$,

$\therefore AC = 2OH = \frac{5}{3}\sqrt{5}$ (1 分)