

九年级期终考试数学试卷参考答案与评分建议

一、选择题：(本大题 6 小题，每小题 4 分，满分 24 分)

1. B; 2. C ; 3. A; 4. D; 5. B; 6. C.

二、填空题：(本大题共 12 题，每题 4 分，满分 48 分)

7. $2\vec{a}-5\vec{b}$; 8. 35; 9. 10; 10. $\frac{3\sqrt{10}}{10}$; 11. 26; 12. 如 $y=-x^2+1$ 等;
13. 1:3 ; 14. $1:\sqrt{2}$; 15. 10 ; 16. 8; 17. $\frac{3}{2}a$; 18. 1:2.

三、解答题：(本大题共 7 题，满分 78 分)

$$\begin{aligned} 19. \text{解: 原式} &= \left(\frac{\sqrt{2}}{2}\right)^2 - \frac{\frac{\sqrt{3}}{3}}{2 \times \frac{\sqrt{3}}{2} + \sqrt{3}} + \frac{1}{2}, \dots\dots\dots (5 \text{ 分}) \\ &= \frac{1}{2} - \frac{1}{6} + \frac{1}{2}, \dots\dots\dots (4 \text{ 分}) \\ &= \frac{5}{6}. \dots\dots\dots (1 \text{ 分}) \end{aligned}$$

20. 解：(1) $\because$ 抛物线 $y=ax^2+bx+c$ 经过点 $A(-1,6)$ 、 $B(1,-2)$ 、 $C(0,1)$ 三点，

$$\therefore \begin{cases} a-b+c=6, \\ a+b+c=-2, \\ c=1. \end{cases} \text{ 解得 } \begin{cases} a=1 \\ b=-4, \\ c=1. \end{cases} \dots\dots\dots (4 \text{ 分})$$

$\therefore$ 抛物线的表达式为 $y=x^2-4x+1$. $\dots\dots\dots (1 \text{ 分})$

抛物线的对称轴 l 为直线 $x=2$. $\dots\dots\dots (1 \text{ 分})$

(2) 过点 B 作 $BH \perp AD$ ，垂足为点 H .

$\because$ 点 A 与点 D 关于对称轴 l 对称，又点 $A(-1,6)$ ， $\therefore D(5,6)$ $\dots\dots\dots (1 \text{ 分})$

$\therefore AD \parallel x$ 轴， $AD=6$. $\because B(1,-2)$ ， $\therefore BH=8$. $\dots\dots\dots (2 \text{ 分})$

$$\therefore S_{\triangle ABD} = \frac{1}{2} \times AD \times BH = \frac{1}{2} \times 6 \times 8 = 24. \dots\dots\dots (1 \text{ 分})$$

$$21. \text{解: (1)} \because AD \parallel BC, \frac{AD}{BC} = \frac{ED}{BE}. \dots\dots\dots (1 \text{ 分})$$

$$\because AD=4, BC=5, \therefore \frac{ED}{BE} = \frac{4}{5}, ED = \frac{4}{9}BD.$$

$$\because \overrightarrow{BC} = \vec{a}, \overrightarrow{BD} = \vec{b}, \therefore \overrightarrow{AD} = \frac{4}{5}\vec{a}, ED = \frac{4}{9}\vec{b}. \dots\dots\dots (2 \text{ 分})$$

$$\because \overrightarrow{AE} = \overrightarrow{AD} - \overrightarrow{ED}, \therefore \overrightarrow{AE} = \frac{4}{5}\vec{a} - \frac{4}{9}\vec{b}. \dots\dots\dots (2 \text{ 分})$$

(2) 方法 1: 过点 A 作 $AF \perp BC$, 垂足为点 F (1 分)

在 $\text{Rt}\triangle ADC$ 中, $AD \perp CD$, $AD=4$, $\tan \angle DAC = \frac{1}{2}$, $\therefore CD=2$ (1 分)

$\because AD \parallel BC$, $\therefore \angle FAD=90^\circ$, 又 $AD \perp CD$, $\therefore$ 四边形 $ADCF$ 是矩形. (1 分)

$\therefore AF=CD=2$, $AD=FC=4$, $\because BC=5$, $\therefore BF=1$, $\therefore AB=\sqrt{5}$ (1 分)

$\therefore \sin \angle ABC = \frac{AF}{AB} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ (1 分)

方法 2: $\because AD \perp CD$, $\tan \angle DAC = \frac{1}{2}$, $\therefore \tan \angle DAC = \frac{CD}{AD} = \frac{1}{2}$, $\because AD=4$, $\therefore CD=2$ (1 分)

$\therefore AC = \sqrt{CD^2 + AD^2}$, $\therefore AC = 2\sqrt{5}$ (1 分)

$\because AD \parallel BC$, $\therefore \angle DAC = \angle ACB$. $\because \frac{AD}{AC} = \frac{AC}{BC} = \frac{2}{\sqrt{5}}$, $\therefore \triangle DAC \sim \triangle ACB$ (2 分)

$\therefore \angle BAC = \angle D = 90^\circ$, $\therefore \sin \angle ABC = \frac{AC}{BC} = \frac{2\sqrt{5}}{5}$ (1 分)

22. 第一次实践 需要测量得到的相关数据有: $NE=b\text{cm}$ (2 分)

利用得到的数据表达树 MN 的高度: $MN = (a+b+40)\text{cm}$ (2 分)

第二次实践 需要测量得到的相关数据有: $EF=c\text{cm}$ (2 分)

解决问题: 设 $MH=x$.

由题意可知 $BC=AB=40\text{cm}$, $BD=20\text{cm}$, $AB \perp CD$, $\therefore \tan \angle CAB = 1$, $\tan \angle D_1A_1B_1 = \frac{1}{2}$.

$\because AH \perp MN$, $\therefore AH = MH = x$. $\tan \angle D_1A_1B_1 = \frac{MH}{A_1H} = \frac{1}{2}$, $\therefore A_1H = 2x$ (2 分)

$\because A_1H - AH = A_1A = B_1B = EF$, $\therefore x = c$ (1 分)

$\therefore MN = (c+a)\text{cm}$ (1 分)

(以上两次实践活动方案合理, 可操作, 答案酌情给分)

23. 证明: (1) $\because AE=AD$, $\therefore \angle E = \angle ADE$ (2 分)

$\because \angle ADE = \angle CDB$, $\therefore \angle E = \angle CDB$ (2 分)

$\because CD$ 平分 $\angle ACB$, $\therefore \angle ACE = \angle BCD$, $\therefore \triangle CEA \sim \triangle CDB$ (2 分)

(2) $\because CF \parallel AE$, $\therefore \angle E = \angle DCF$ (1 分)

$\because \angle DCF = \angle DCB + \angle BCF$, $\angle E = \angle CDF = \angle ACE + \angle CAD$, $\therefore \angle BCF = \angle CAD$ (1 分)

又 $\angle F = \angle F$, $\therefore \triangle CFB \sim \triangle AFC$, $\therefore \frac{BF}{CF} = \frac{BC}{AC}$ (2 分)

由 $\triangle CEA \sim \triangle CDB$, $\therefore \frac{BC}{AC} = \frac{BD}{AE}$. 又 $AE=AD$, $\therefore \frac{BC}{AC} = \frac{BD}{AD}$, $\therefore \frac{BD}{AD} = \frac{BF}{CF}$ (2 分)

24. (1) $C(0, c)$, $P(2, c+4)$, (2 分)

$$D\left(-\frac{c}{2}, 0\right) . \dots\dots\dots (2 \text{ 分})$$

(2) ①方法 1: 过点 P' 作 $P'H \perp y$ 轴, 垂足为点 H .

由题意可得直 $PC: y = 2x + c$, 设 $P'(m, 2m + c)$. $\dots\dots\dots (1 \text{ 分})$

$\therefore$ 新抛物线 $y = -x^2 + 2mx - m^2 + 2m + c$, $\therefore E(0, -m^2 + 2m + c)$. $\dots\dots\dots (1 \text{ 分})$

$\because x$ 轴 $\perp y$ 轴, $\therefore \angle P'EC = \angle PDO$, $\therefore \tan \angle P'EC = \tan \angle PDO = 2$,

$$\therefore \tan \angle P'EC = \frac{P'H}{HE} = \frac{-m}{2m + c + m^2 - 2m - c} = 2, \therefore m = -\frac{1}{2}, \therefore P'\left(-\frac{1}{2}, c - 1\right) . \dots\dots\dots (1 \text{ 分})$$

$\therefore$ 该抛物线向左平移 $\frac{5}{2}$ 个单位, 向下平移 5 个单位. $\dots\dots\dots (1 \text{ 分})$

方法 2: 过点 P' 作 $P'H \perp y$ 轴, 垂足为点 H .

设 $P'(m, k)$, $\therefore$ 新抛物线 $y = -x^2 + 2mx - m^2 + k$,

$\therefore E(0, -m^2 + k)$. $\dots\dots\dots (1 \text{ 分})$

$\because x$ 轴 $\perp y$ 轴, $\therefore \angle P'EC = \angle PDO$, $\therefore \tan \angle P'EC = \tan \angle PDO = 2$,

$$\therefore \tan \angle P'EC = \frac{P'H}{HE} = \frac{-m}{k + m^2 - k} = 2, \therefore m = -\frac{1}{2} . \dots\dots\dots (1 \text{ 分})$$

$$\text{又 } \frac{P'H}{CH} = \frac{1}{2}, \therefore \frac{\frac{1}{2}}{c - k} = \frac{1}{2}, \therefore k = c - 1, \therefore P'\left(-\frac{1}{2}, c - 1\right) . \dots\dots\dots (1 \text{ 分})$$

$\therefore$ 该抛物线向左平移 $\frac{5}{2}$ 个单位, 向下平移 5 个单位. $\dots\dots\dots (1 \text{ 分})$

②方法 1: 由 DA' 被 y 轴平分, $D\left(-\frac{c}{2}, 0\right)$, 设 $A'\left(\frac{c}{2}, y\right)$, $\therefore A\left(\frac{c}{2} + \frac{5}{2}, 0\right)$. $\dots\dots\dots (2 \text{ 分})$

$$\because \text{点 } A \text{ 在原抛物线上}, \therefore -\left(\frac{c+5}{2}\right)^2 + 4 \times \frac{c+5}{2} + c = 0 . \dots\dots\dots (1 \text{ 分})$$

$$\text{解得 } c_1 = 5, c_2 = -3 (\text{舍}) \therefore y = -x^2 + 4x + 5 . \dots\dots\dots (1 \text{ 分})$$

方法 2: 由 $y = -x^2 + 4x + c$, 令 $y = 0$, 得 $x = 2 \pm \sqrt{c+4}$, $\therefore A(2 + \sqrt{c+4}, 0)$. $\dots\dots\dots (1 \text{ 分})$

$$\therefore A'\left(\sqrt{c+4} - \frac{1}{2}, -5\right) . \dots\dots\dots (1 \text{ 分})$$

过点 A' 作 $A'G \parallel x$ 轴交 y 轴于点 G .

$$\because A'D \text{ 被 } y \text{ 轴平分}, \therefore DO = A'G, \therefore \frac{c}{2} = \sqrt{c+4} - \frac{1}{2}, \dots\dots\dots (1 \text{ 分})$$

$$\text{解得 } c_1 = 5, c_2 = -3 (\text{舍}) \therefore y = -x^2 + 4x + 5 . \dots\dots\dots (1 \text{ 分})$$

25. 解: (1) 过点 C 作 $CG \perp AB$, 垂足为点 G .

$$\because BC=5, \sin B = \frac{4}{5}, \therefore BG=3, CG=4. \dots\dots\dots (1 \text{ 分})$$

$$\because AB=9, \therefore AG=6, \because \angle AGC=90^\circ, \therefore \tan \angle BAC = \frac{CG}{AG} = \frac{2}{3}. \dots\dots\dots (1 \text{ 分})$$

$$\because \angle CPF=90^\circ, \frac{FP}{PC} = \frac{2}{3}, \therefore \tan \angle PCF = \tan \angle BAC = \frac{2}{3}. \dots\dots\dots (1 \text{ 分})$$

$$\because \angle BAC = \angle PCF < 90^\circ, \therefore \angle BAC = \angle PCF. \dots\dots\dots (1 \text{ 分})$$

$$(2) \because AB \parallel CD, \therefore \angle BAC = \angle ACD.$$

$$\because \angle PCF = \angle BAC, \therefore \angle PCF = \angle ACD, \therefore \angle PCA = \angle FCD. \dots\dots\dots (1 \text{ 分})$$

$$\because \triangle APC \sim \triangle EFC, \angle PEC = \angle APE < 90^\circ, \therefore \angle APC = \angle EFC, \therefore \angle BPC = \angle PFC. \dots\dots (1 \text{ 分})$$

$$\therefore \cot \angle BPC = \cot \angle PFC, \therefore \frac{PG}{CG} = \frac{2}{3}, \dots\dots\dots (1 \text{ 分})$$

$$\therefore PG = \frac{8}{3}, \therefore BP = \frac{17}{3}. \dots\dots\dots (1 \text{ 分})$$

(3) 过点 H 作 $HM \perp AB$, 垂足为点 M .

$$\because \angle FPC=90^\circ, \therefore \angle MPH = \angle PCG.$$

$$\text{又 } \angle HMP = \angle CGP = 90^\circ, \therefore \triangle MPH \sim \triangle GCP. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{MP}{GC} = \frac{MH}{GP} = \frac{PH}{CP}, \therefore \frac{S_{\triangle HFC}}{S_{\triangle PHC}} = \frac{1}{3}, \therefore \frac{FH}{PH} = \frac{1}{3}. \dots\dots\dots (1 \text{ 分})$$

$$\text{①当点 } F \text{ 在线段 } PH \text{ 的延长线上时, 由 } \frac{PF}{PC} = \frac{2}{3}, \text{ 可得 } \frac{PH}{PC} = \frac{1}{2}. \dots\dots\dots (1 \text{ 分})$$

$$\text{设 } MH=a, \therefore GP=2a, MP=2, AM = \frac{3}{2}a, \therefore 2a + \frac{3}{2}a + 2 + 3 = 9, a = \frac{8}{7}.$$

$$\therefore \frac{AH}{AC} = \frac{MH}{CG} = \frac{2}{7}. \dots\dots\dots (1 \text{ 分})$$

$$\text{②当点 } F \text{ 在线段 } PH \text{ 上时, 可得 } PH=PC. \dots\dots\dots (1 \text{ 分})$$

$$\text{设 } MH=b, \therefore GP=b, MP=4, AM = \frac{3}{2}b, \therefore b + \frac{3}{2}b + 4 + 3 = 9, b = \frac{4}{5}.$$

$$\therefore \frac{AH}{AC} = \frac{MH}{CG} = \frac{1}{5}. \dots\dots\dots (1 \text{ 分})$$

综上所述 $\frac{AH}{AC}$ 的值为 $\frac{2}{7}$ 或 $\frac{1}{5}$.