

嘉定区 2024 学年第一学期九年级期终学业质量调研测试

数学试卷参考答案及评分标准

一、 1. C; 2. B; 3. B; 4. B; 5. D; 6. D .

二、 7. $-\frac{1}{4}$; 8. $a > 2$; 9. $(4,0)$; 10. $<$; 11. $x = -1$; 12. $\frac{4}{9}$; 13. $\frac{1}{3}\vec{b} - \frac{1}{3}\vec{a}$;

14. $\frac{4\sqrt{2}}{9}$; 15. 18; 16. 14; 17. $4 + 2\sqrt{5}$; 18. $12\sqrt{3}$.

三、 19. 解: $\sin^2 45^\circ - \frac{2 \cos 60^\circ}{\cot 30^\circ - \tan 45^\circ}$
 $= \left(\frac{\sqrt{2}}{2}\right)^2 - \frac{2 \times \frac{1}{2}}{\sqrt{3} - 1}$ (8 分)
 $= \frac{1}{2} - \frac{\sqrt{3} + 1}{2}$ (1 分)
 $= -\frac{\sqrt{3}}{2}$ (1 分)

20. (1) 解: ② (2 分)

$\because$ 抛物线 $y = -\frac{1}{3}x^2 + bx + c$ 的顶点是 $(1, \frac{4}{3})$,

$\therefore$ 它的表达式为 $y = -\frac{1}{3}(x-1)^2 + \frac{4}{3}$, (2 分)

化为一般式 $y = -\frac{1}{3}x^2 + \frac{2}{3}x + 1$, 得 $b = \frac{2}{3}$, $c = 1$ (2 分)

(2) 解: 由 $-\frac{1}{3}(x-1)^2 + \frac{4}{3} = 0$ 解得 $x_1 = -1, x_2 = 3$, 得 $A(3,0)$, (2 分)

过点 $D(1, \frac{4}{3})$ 作 x 轴的垂线, 垂足为 H ,

则 $DH = \frac{4}{3}$, $AH = 3 - 1 = 2$, (1 分)

在 $Rt\triangle AHD$ 中, $\angle AHD = 90^\circ$, $\tan \angle DAO = \frac{DH}{AH} = \frac{2}{3}$ (1 分)

21. (1) 解: $\because$ 在 $\triangle ABD$ 和 $\triangle CBA$ 中

$\begin{cases} \angle BAD = \angle C \\ \angle B = \angle B \end{cases}$
 $\therefore \triangle ABD \sim \triangle CBA$ (2 分)

$\therefore \frac{AB}{CB} = \frac{BD}{BA}$ (1 分)

$\because BD = 12, CD = 15, \therefore BC = BD + CD = 27$ (1 分)

$\therefore \frac{AB}{27} = \frac{12}{BA}$, 可得 $AB = 18$ (1 分)

(2) $\because \angle BAD = \angle C, \angle ADE = \angle C \therefore \angle BAD = \angle ADE$

$\therefore AB \parallel DE$ (1 分)

$\therefore \frac{DE}{AB} = \frac{CD}{CB} = \frac{15}{27}$, 可得 $DE = 10$ (1 分)

如图 1, 作 $EH \perp BC$, 垂足为点 H ,

$\because AB \parallel DE \therefore \angle EDH = \angle B = 60^\circ$ (1 分)

在 $Rt\triangle EDH$ 中, $EH = DE \cdot \sin \angle EDH = 5\sqrt{3}$ (1 分)

$\therefore S_{\triangle EDC} = \frac{1}{2} CD \cdot EH = \frac{75}{2}\sqrt{3}$ (1 分)

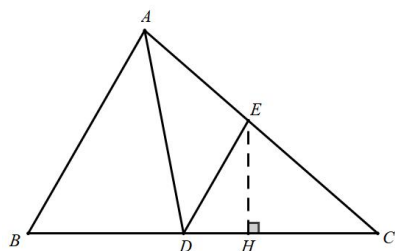


图 1

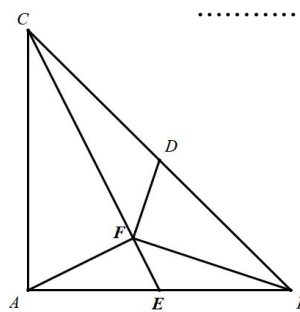


图 2

22. (1) 证明: $\because \angle BAC = 90^\circ, AF \perp CE \therefore \angle EFA = \angle EAC = 90^\circ$ (1 分)

又 $\because \angle AEF = \angle CEA, \therefore \triangle EAF \sim \triangle ECA$ (1 分)

$\therefore \frac{EA}{EC} = \frac{EF}{EA}$ (1 分)

$\because$ 点 E 是边 AB 的中点, 即: $EA = EB, \therefore \frac{EB}{EC} = \frac{EF}{EB}$ (1 分)

又 $\because \angle BEF = \angle CEB$ (1 分)

$\therefore \triangle EFB \sim \triangle EBC$ (1 分)

(2) 证明: 如图 2, 由 (1) 知: $\triangle EFB \sim \triangle EBC$

$\therefore \angle EBF = \angle ECB, \angle EFB = \angle EBC$ (1 分)

$\because \angle BAC = 90^\circ, AB = AC, \therefore \angle CBA = 45^\circ$

$\therefore \angle EFB = \angle EBC = 45^\circ$

又 $\because \angle BFA = \angle AFE + \angle EFB = 135^\circ, \angle CFB = 180^\circ - \angle EFB = 135^\circ$

$\therefore \angle BFA = \angle CFB$ (1 分)

又由 $\angle ABF = \angle BCF \therefore \triangle BFA \sim \triangle CFB$ (1 分)

$\because$ 点 E 是边 AB 的中点, 点 D 是边 BC 的中点, $\therefore \frac{DF}{EF} = \frac{BC}{AB}$ (1 分)

$$\therefore \text{在 Rt} \triangle ABC \text{ 中, } \cos \angle ABC = \frac{AB}{BC} = \frac{\sqrt{2}}{2}, \text{ 即: } \frac{BC}{AB} = \sqrt{2} \quad \dots\dots (1 \text{ 分})$$

$$\therefore \frac{DF}{EF} = \frac{BC}{AB} = \sqrt{2} \quad \dots\dots\dots (1 \text{ 分})$$

23. (1) 解: 如图 3, 过点 N 作直线 MM' 的垂线, 垂足为点 H ,

$$\therefore \angle PMN = 90^\circ, \angle M'OP = \alpha, \therefore \angle NMH = 90^\circ - \alpha,$$

$$\text{又} \therefore \angle H = 90^\circ, \therefore \angle MNH = 90^\circ - \angle NMH = \alpha, \quad \dots\dots\dots (1 \text{ 分})$$

$$\text{在 Rt} \triangle MHN \text{ 中, } \angle MHN = 90^\circ, \cos \angle MNH = \frac{NH}{MN} \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore MN = m$$

$$\therefore \text{点 } N \text{ 到直线 } MM' \text{ 的距离 } NH = m \cos \alpha. \quad \dots\dots\dots (2 \text{ 分})$$

(也可以是 $NH = m \sin(90^\circ - \alpha)$)

(2) 解: $\therefore \sin \alpha \approx 0.8, \cos \alpha \approx 0.6,$

$$\text{由 (1) 得 } NH = m \cos \alpha \approx 0.6m, \text{ 同理可得 } MH = m \sin \alpha \approx 0.8m, \quad \dots\dots (1 \text{ 分})$$

如图 3, 过点 N' 作 MM' 的垂线, 垂足为点 G ,

$$\therefore MP \parallel M'N', \therefore \angle N'M'G = \angle M'MP = \alpha, \quad \dots\dots\dots (1 \text{ 分})$$

在 $\text{Rt} \triangle M'GN'$ 中, $\angle M'GN' = 90^\circ,$

$$\therefore M'N' = 5, \therefore M'G = 5 \times \cos \alpha \approx 3, N'G = 5 \times \sin \alpha \approx 4, \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore OM' = 23, OM = 132, \therefore GO = OM' - M'G = 23 - 3 = 20, \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \angle N'GO = \angle NHO = 90^\circ, \therefore N'G \parallel NH$$

$$\therefore \frac{N'G}{NH} = \frac{GM}{HM} \quad \dots\dots\dots (1 \text{ 分})$$

$$\text{可得方程 } \frac{4}{0.6m} = \frac{20}{132 + 0.8m}, \text{ 解得 } m = 60, \text{ 所以 } MN = 60. \quad \dots\dots\dots (1 \text{ 分})$$

24. (1) 解: $\therefore$ 抛物线 $y = ax^2 + bx - 1$ 经过点 $(2,3)$ 和点 $(-4,3)$,

$$\therefore \begin{cases} 4a + 2b - 1 = 3, \\ 16a - 4b - 1 = 3. \end{cases} \quad \dots\dots\dots (2 \text{ 分})$$

$$\text{解得 } \begin{cases} a = \frac{1}{2}, \\ b = 1 \end{cases}, \therefore \text{抛物线的表达式为 } y = \frac{1}{2}x^2 + x - 1 \quad \dots\dots\dots (2 \text{ 分})$$

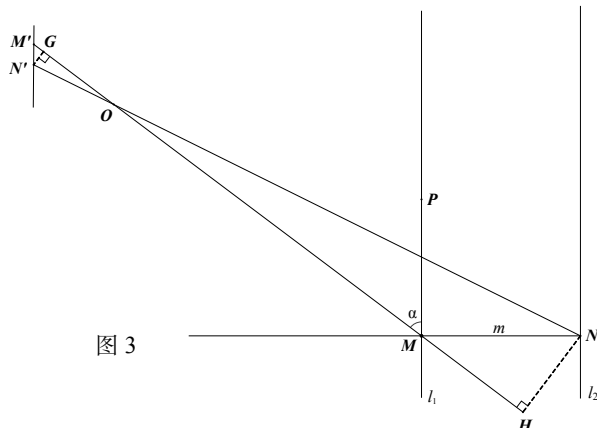


图 3

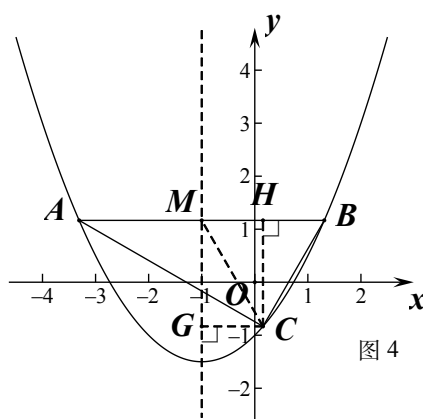


图 4

(2) 解: ① 抛物线的对称轴是直线 $x = -1$,

如图 4, 联结 CM , 作 $CG \perp$ 直线 $x = -1$, $CH \perp AB$,

$\because AB \parallel x$ 轴, $\therefore$ 四边形 $MGCH$ 是矩形, 点 M 是 AB 的中点, (1 分)

$\because$ 点 C 到抛物线对称轴的距离为 t , 所以 $CG = MH = t$, (1 分)

$\because \angle ACB = 90^\circ$, $\therefore CM = BM$,

$\because \angle BAC = 30^\circ$, $\therefore \angle B = 60^\circ$, $\triangle BMC$ 是等边三角形,

$\therefore CB = CM$, $MH = HB$, $\therefore BM = MH + HB = 2t$, (1 分)

$\therefore$ 点 B 的横坐标是 $2t - 1$ (1 分)

② $\because \angle BHC = 90^\circ$, $\angle B = 60^\circ$, $BH = t$,

$\therefore CH = BH \cdot \tan B = \sqrt{3} \cdot t$, (1 分)

即点 B 的纵坐标比点 C 的纵坐标大 $\sqrt{3} \cdot t$, 因为点 C 的横坐标是 $t - 1$,

可得 $\frac{1}{2}(t-1)^2 + (t-1) - 1 + \sqrt{3} \cdot t = \frac{1}{2}(2t-1)^2 + (2t-1) - 1$, (1 分)

解得 $t_1 = 0$ (点 A, B, C 重合, 舍), $t_2 = \frac{2}{3}\sqrt{3}$, (1 分)

$\therefore$ 点 C 的横坐标是 $\frac{2}{3}\sqrt{3} - 1$ (1 分)

25. (1) 证明: $\because AB = AC$, $CD = CE$

$\therefore \angle B = \angle ACB$, $\angle CDE = \angle CED$ (1 分)

$\because CD \perp AB$ $\therefore \angle CDB = 90^\circ$

$\therefore \angle B + \angle BCD = 90^\circ$, 即: $\angle B = 90^\circ - \angle BCD$ (1 分)

$\therefore \angle DCE = \angle ACB - \angle BCD = \angle B - \angle BCD = 90^\circ - 2\angle BCD$ (1 分)

$\because \angle CDE + \angle CED + \angle DCE = 180^\circ$

$\therefore \angle DCE = 180^\circ - \angle CDE - \angle CED = 180^\circ - 2\angle CDE$ (1 分)

$$\therefore 90^\circ - 2\angle BCD = 180^\circ - 2\angle CDE$$

$$\therefore \angle CDE = \angle BCD + 45^\circ \quad \dots\dots\dots (1 \text{ 分})$$

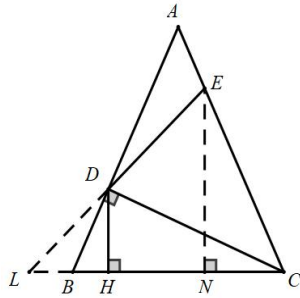


图 5

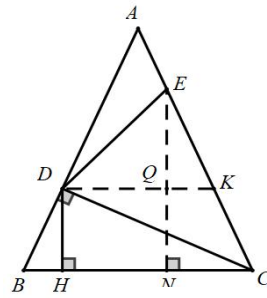


图 6

(2) 证明: 【法 1】如图 5, 分别延长 ED 、 CB 相交于点 L .

由 (1) 知: $\angle CDE = \angle BCD + 45^\circ$

$$\therefore \angle CDE = \angle BCD + \angle L, \therefore \angle L = 45^\circ \quad \dots\dots\dots (1 \text{ 分})$$

过点 E 作 $EN \perp BC$, 垂足为点 N .

$$\text{在 Rt}\triangle ENL \text{ 中, } EL = \frac{EN}{\sin \angle L} = \frac{EN}{\sin 45^\circ} = \sqrt{2}EN \quad \dots\dots\dots (1 \text{ 分})$$

$$\text{在 Rt}\triangle DHL \text{ 中, } DL = \frac{DH}{\sin \angle L} = \frac{DH}{\sin 45^\circ} = \sqrt{2}DH$$

$$\therefore DE = EL - DL = \sqrt{2}(EN - DH) \quad \dots\dots\dots (1 \text{ 分})$$

$\therefore$ 在 $\text{Rt}\triangle CDB$ 中, $\angle BCD + \angle CBD = 90^\circ$

在 $\text{Rt}\triangle CDH$ 中, $\angle BCD + \angle CDH = 90^\circ$

$\therefore \angle CDH = \angle CBD$, 从而 $\angle CDH = \angle ECN$

又 $\because \angle CHD = \angle ENC = 90^\circ$, $CD = CE$, $\therefore \triangle CMD \cong \triangle ENC$ $\dots\dots\dots (1 \text{ 分})$

$$\therefore EN = CH, \therefore DE = \sqrt{2}(EN - DH) = \sqrt{2}(CH - DH) \quad \dots\dots\dots (1 \text{ 分})$$

【法 2】如图 6, 过点 D 作 $DK \parallel BC$, 交 AC 于点 K . (思路同上, 根据步骤酌情赋分)

(3) 解: $\because$ 点 F 是 CD 的中点 $\therefore CF = DF$

由 $\triangle CFG$ 与 $\triangle BCD$ 相似

①当 $\angle CFG = \angle CDB = 90^\circ$ 时, 则 $\triangle CFG \sim \triangle CDB$.

$\therefore EG \perp CD$, 从而 EG 垂直平分 CD .

$\therefore ED = EC$, 又 $CD = CE$

$\therefore \triangle CDE$ 是等边三角形, 从而 $\angle CDE = \angle BCD + 45^\circ = 60^\circ$

$$\therefore \angle BCD = 15^\circ, \angle B = 75^\circ \quad \dots\dots\dots (1 \text{ 分})$$

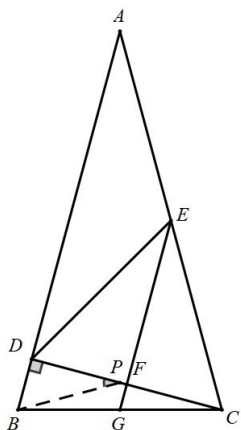


图 7

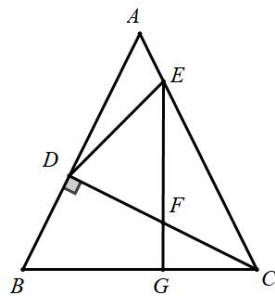


图 8

如图 7, 在 $\angle CBD$ 内部作 $\angle DBP = 60^\circ$, 射线 BP 交 CD 边于点 P
 $\therefore \angle PBC = \angle PCB = 15^\circ$, 从而 $PB = PC$.

在 $\text{Rt}\triangle PDB$ 中, 设 $DB = x$, 则 $DP = \sqrt{3}x$, $BP = PC = 2x$

$$\therefore k = \tan \angle CBD = \frac{CD}{BD} = \frac{2x + \sqrt{3}x}{x} = 2 + \sqrt{3} \quad \dots\dots\dots (1 \text{ 分})$$

②当 $\angle CGF = \angle CDB = 90^\circ$ 时, 如图 8, 则 $\triangle CGF \sim \triangle CDB$.

$\therefore \angle CFG = \angle CBD$, 从而 $\angle CFG = \angle ECG$.

又 $\because \angle CGF = \angle EGC$, $\therefore \triangle CFG \sim \triangle ECG$. $\dots\dots\dots (1 \text{ 分})$

$$\therefore \frac{CG}{FG} = \frac{EC}{CF} = \frac{CD}{CF} = 2$$

$$\therefore k = \tan \angle CBD = \tan \angle CFG = \frac{CG}{FG} = 2. \quad \dots\dots\dots (1 \text{ 分})$$

综上所述, $k = 2 + \sqrt{3}$ 或 $k = 2$.