

2024 年松江区模拟考试试卷九年级数学参考答案

一、选择题 (本大题共 6 题, 每题 4 分, 满分 24 分)

1. A 2. C 3. D 4. A 5. D 6. C

二、填空题 (本大题共 12 题, 每题 4 分, 满分 48 分)

7. $\sqrt{2}$; 8. $a(a-1)$; 9. $1 \leq x < 2$; 10. $-\frac{1}{4}$; 11. 增大; 12. 13.2; 13. $\frac{1}{3}$; 14.

$y = x^2 - x$ (答案不唯一); 15. $\frac{1}{3}a + \frac{2}{3}b$; 16. 240; 17. 12.5; 18. 5 或 $\frac{13}{2}$.

三、解答题 (本大题共 7 题, 满分 78 分)

19. 计算: $\left(1\frac{7}{9}\right)^{\frac{1}{2}} + |2 - \sqrt{3}| + \frac{4}{\sqrt{3}-1} - \left(\frac{\sqrt{3}}{3}\right)^{-1}$.

$$\begin{aligned} \text{原式} &= \frac{4}{3} + (2 - \sqrt{3}) + (2\sqrt{3} + 2) - \sqrt{3} \\ &= \frac{4}{3} + 2 - \sqrt{3} + 2\sqrt{3} + 2 - \sqrt{3} \\ &= 5\frac{1}{3} \end{aligned}$$

20. 解方程组: $\begin{cases} x^2 - 3xy + 2y^2 = 0, & \text{①} \\ x^2 + y^2 = 5. & \text{②} \end{cases}$

解: 由方程①得 $(x-y)(x-2y) = 0$, 得到 $x-y=0$ 或 $x-2y=0$.

将它们与方程②分别组成方程组, 得

$$(I) \begin{cases} x-y=0, \\ x^2+y^2=5. \end{cases} \quad \text{或} \quad (II) \begin{cases} x-2y=0, \\ x^2+y^2=5. \end{cases}$$

$$\text{解方程组 (I), 得} \begin{cases} x_1 = \frac{\sqrt{10}}{2}, \\ y_1 = \frac{\sqrt{10}}{2}; \end{cases} \quad \begin{cases} x_2 = -\frac{\sqrt{10}}{2}, \\ y_2 = -\frac{\sqrt{10}}{2}. \end{cases}$$

$$\text{解方程组 (II), 得} \begin{cases} x_3 = 2, \\ y_3 = 1; \end{cases} \quad \begin{cases} x_4 = -2, \\ y_4 = -1. \end{cases}$$

$$\text{所以原方程组的解是} \begin{cases} x_1 = \frac{\sqrt{10}}{2}, \\ y_1 = \frac{\sqrt{10}}{2}; \end{cases} \quad \begin{cases} x_2 = -\frac{\sqrt{10}}{2}, \\ y_2 = -\frac{\sqrt{10}}{2}; \end{cases} \quad \begin{cases} x_3 = 2, \\ y_3 = 1; \end{cases} \quad \begin{cases} x_4 = -2, \\ y_4 = -1. \end{cases}$$

21. (1) 联结 AO , 设 $AO=BO=r$

$$\because BC=8, \therefore CO=8-r$$

$$\because \angle C=90^\circ, AC=4$$

$$\therefore 4^2 + (8-r)^2 = r^2$$

$$r=5$$

$\therefore \odot O$ 的半径长是 5.

(2) 过点 P 作 $PH \perp AB$, 垂足为 H .

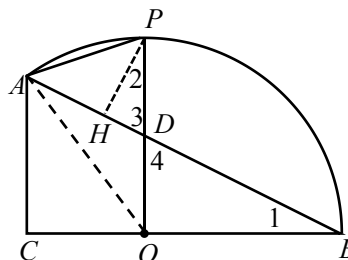
$$\because \tan \angle 1 = \frac{OD}{OB} = \frac{AC}{BC} = \frac{1}{2}, \therefore OD = \frac{5}{2}, PD = \frac{3}{2}, BD = \frac{5}{2}\sqrt{5}.$$

$$\text{又} \because \angle 1 + \angle 4 = \angle 2 + \angle 4 = 90^\circ \quad \angle 3 = \angle 4 \quad \therefore \angle 1 = \angle 2$$

$$\therefore \sin \angle 2 = \frac{DH}{PD} = \sin \angle 1 = \frac{OD}{BD} = \frac{\sqrt{5}}{5}.$$

$$\therefore HD = \frac{\sqrt{5}}{2}, PH = \sqrt{5}, AH = \sqrt{5}$$

$$\therefore \tan \angle PAB = \frac{PH}{AH} = \frac{\sqrt{5}}{\sqrt{5}} = 1.$$



(图 5)

22. (1) ① $AD \perp BC$, AD 平分 BC ;

② 四边形 $ABDC$ 是轴对称图形, 直线 AD 所在的直线是它的对称轴;

③ $\angle BAC=60^\circ$, $\angle ABD=\angle ACD=75^\circ$, $\angle BDC=150^\circ$.

(2) 画图

$\because$ 在菱形 $ABCD$ 中, 且 $AB=BC=CD=DA=BD$

$\therefore \angle BAD=60^\circ$, $\angle BAC=30^\circ$, $AC \perp BD$

$$\therefore \cos \angle BAC = \cos 30^\circ = \frac{AO}{AB} = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{AC}{AB} = \frac{2AO}{AB} = \sqrt{3}.$$

(3) 画图

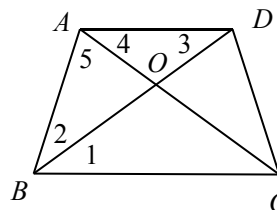
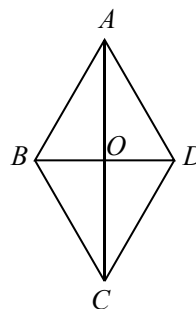
$\because AB=AD=CD \quad BD=AC=BC$

$\therefore \angle 2 = \angle 3 = \angle 1 = \angle 4$

$\angle 5 = 2\angle 1$

$\because AD \parallel BC \quad \therefore \angle DAB + \angle ABC = 180^\circ \quad 5\angle 1 = 180^\circ$

$\therefore \angle ABC = \angle BCD = 72^\circ \quad \angle BAD = \angle ADC = 108^\circ.$



23. (1) 分别过 O_1, O_2 作 $O_1M \perp AD, O_2N \perp AP$, 垂足分别为 M, N .

$$\because O_1O_2 \perp AB, AD=AB$$

$$\therefore O_1M=O_1C,$$

$$\therefore \angle O_1AD=\angle O_1AB$$

同理 $\therefore \angle O_2AP=\angle O_2AB$

$$\therefore \angle O_1AO_2 = \frac{1}{2}\angle DAB + \frac{1}{2}\angle PAB = \frac{1}{2}(\angle DAB + \angle PAB) = \frac{1}{2} \times 180^\circ = 90^\circ$$

$$\therefore O_1A \perp O_2A$$

(2) 分别过 O_1, O_2 作 $O_1M \perp AD, O_2N \perp AP$, 垂足分别为 M, N .

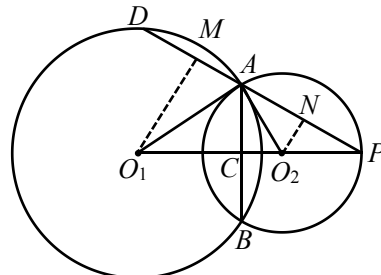
$$\therefore DM=AM=\frac{1}{2}AD, AN=PN=\frac{1}{2}AP$$

$$\because O_1M \parallel O_2N, \therefore \frac{PO_2}{PO_1} = \frac{PN}{PM}$$

$$\therefore PO_1=3PO_2, \therefore PM=3PN$$

$$\therefore AP=2PN, \therefore AM=AN=PN,$$

$$\therefore AD=AP$$



(图 7)

24.(1) $\because$ 抛物线过点 $A(2,0) \therefore -4+2b+c=0, c=4-2b$

$$\therefore -\frac{b}{2a} = -\frac{b}{2 \times (-1)} = \frac{b}{2}, \frac{4ac-b^2}{4a} = \frac{b^2-8b+16}{4} \therefore C \left(\frac{b}{2}, \frac{b^2-8b+16}{4} \right)$$

$$\because B(0, 2) \therefore \text{直线 } AB: y=-x+2$$

顶点 C 在线段 AB 上, $\therefore \frac{b^2-8b+16}{4} = -\frac{b}{2} + 2$ 得 $b=4$ (舍去) 或 $b=2$

$$\therefore c=4-2 \times 2=0$$

$$\therefore b=2, c=0.$$

$$(2) \textcircled{1} y = -x^2 + 2x = -(x-1)^2 + 1$$

$\therefore$ 对称轴为直线 $x=1$, 顶点 C 为 $(1,1)$

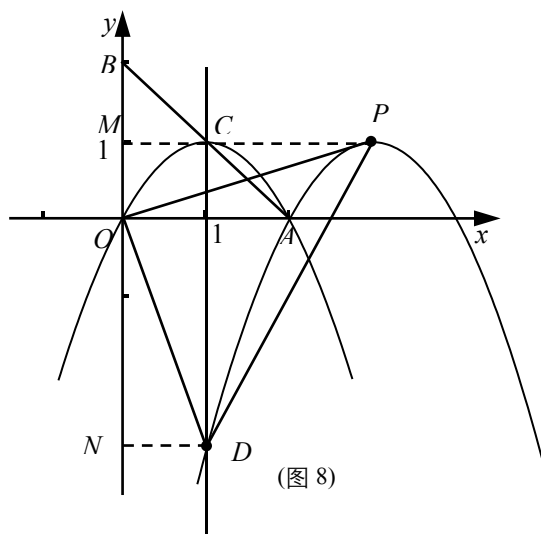
当 $m=2$ 时,

$$y = -(x-3)^2 + 1$$

顶点 $P(3,1)$

当 $x=1$ 时, $y=-3$

$$\therefore D(1,-3)$$



(图 8)

过点 P, D , 作 y 轴垂线 垂足分别为 M, N

$$S_{\triangle ODP} = S_{\text{梯}PMND} - S_{\triangle OMP} - S_{\triangle ODN} = \frac{1}{2} \times (1+3) \times 4 - \frac{1}{2} \times 3 \times 1 \times 2 = 5$$

$$\textcircled{2} \because EC=EP \therefore EP=ED$$

$$\because AF \parallel CP \therefore CF=FD=1 \therefore D(1, -1)$$

设平移后的解析式为 $y = -(x-1-m)^2 + 1$

$$\therefore -1 = -(-m)^2 + 1 \therefore m = \sqrt{2} \quad (m > 0)$$

25. (1) $\because PE \perp AC$, F 为 AP 的中点

$$\therefore AE=EF$$

$$\therefore \angle 1 = \angle 2$$

$\because$ 在矩形 $ABCD$ 中, $\therefore \angle BAD = 90^\circ$

$$\therefore \angle 3 = \angle 4,$$

$$\therefore AB=BE$$

(2) PF 的长度不变

$$\because AD \parallel BC \therefore \angle 1 = \angle 5 \therefore \tan \angle 1 = \frac{PE}{AE} = \tan \angle 5 = \frac{AB}{BC} = \frac{1}{2}$$

$$\because \angle 1 + \angle 3 = \angle 1 + \angle 6 = 90^\circ \therefore \angle 3 = \angle 6 \quad \text{又} \angle 4 = \angle 7 \therefore \triangle EPF \sim \triangle EAB$$

$$\therefore \frac{PF}{AB} = \frac{PE}{AE} = \frac{1}{2} \quad AB=1, \therefore PF = \frac{1}{2}.$$

(3) 联结 FG , 过点 P 作 $PH \perp BC$, 垂足为 H

$$\because \cot \angle 8 = \frac{GH}{PH} = \tan \angle 5 = \frac{1}{2}, PH=AB=1$$

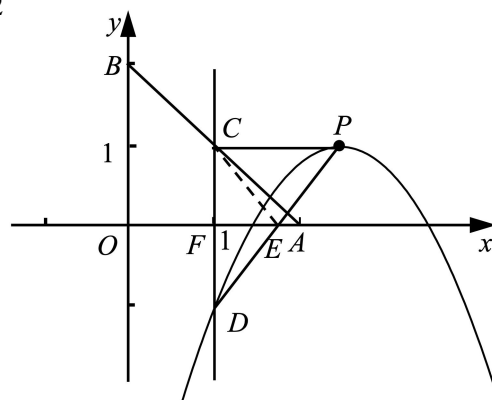
$$\therefore GH = PF = \frac{1}{2}$$

$$\therefore \text{四边形} PFGH \text{ 是矩形} \therefore \angle 1 + \angle 6 = \angle 10 + \angle 6 = 90^\circ \therefore \angle 1 = \angle 10$$

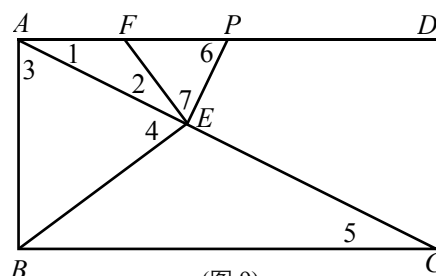
当 $\angle AFE = \angle FEG$ 时 (均为钝角), $\triangle EFG \sim \triangle EFA$

$$\therefore \angle 9 = \angle 7, \therefore PE = PF = \frac{1}{2}$$

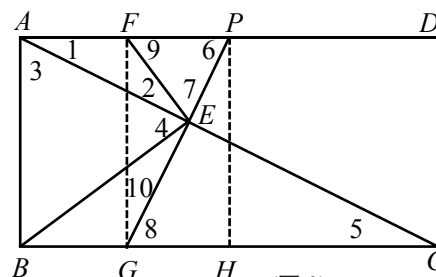
$$\therefore AP = \frac{\sqrt{5}}{2}$$



(图 8)



(图 9)



(图 9)