

青浦区 2022-2023 学年第一学期期终学业质量调研 九年级数学试卷

参考答案及评分说明 Q2023.2

一、选择题：(本大题共 6 题，每小题 4 分，满分 24 分)

1. D; 2. C; 3. C; 4. B; 5. B; 6. A.

二、填空题：(本大题共 12 题，每小题 4 分，满分 48 分)

7. $\frac{5}{3}$; 8. $-2\vec{e}$; 9. 1:2; 10. 2; 11. 上升;

12. $y=-(x+1)^2$;

13. 2; 14. 15; 15. 20; 16. $\frac{5}{2}$; 17. $\frac{16\sqrt{2}}{7}$; 18. $\frac{5}{14}$.

三、解答题：

19. 解：原式 $= 2 \times \frac{1}{2} + \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{3}}{3}\right)^{-1} + \sqrt{(1-\sqrt{3})^2}$ (4 分)

$= 1 + \frac{1}{2} - \sqrt{3} + \sqrt{3} - 1$ (4 分)

$= \frac{1}{2}$ (2 分)

20. 解：(1) $\because$ 四边形 $ABCD$ 是平行四边形,
 $\therefore AB \parallel CD, AD \parallel BC, AB = CD$ (1 分)

$\therefore \frac{AE}{AB} = \frac{AE}{CD} = \frac{AF}{FD}$ (2 分)

$\because DF = 2AF, \therefore \frac{AF}{FD} = \frac{1}{2}$ (1 分)

$\therefore \frac{EA}{AB} = \frac{1}{2}$ (1 分)

(2) $\because$ 四边形 $ABCD$ 是平行四边形, $\therefore AD \parallel BC, AD = BC$ (1 分)

$\because DF = 2AF, \therefore \frac{DF}{AD} = \frac{DF}{BC} = \frac{2}{3}$ (1 分)

$\because \overrightarrow{BA} = \vec{a}, \overrightarrow{BC} = \vec{b}, \therefore \overrightarrow{CD} = \vec{a}, \overrightarrow{DF} = -\frac{2}{3}\vec{b}$ (2 分)

$\therefore \overrightarrow{CF} = \overrightarrow{CD} + \overrightarrow{DF} = \vec{a} - \frac{2}{3}\vec{b}$ (1 分)

21. 解：(1) $\because AD \perp BC, AD = 4, \sin \angle C = \frac{2\sqrt{5}}{5}$,

$$\therefore \frac{AD}{AC} = \frac{4}{AC} = \frac{2\sqrt{5}}{5}, \quad \therefore AC = 2\sqrt{5}. \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore \text{在 Rt}\triangle ACD \text{ 中, } CD = \sqrt{AC^2 - AD^2} = 2. \\ \therefore BC = 5, \quad \therefore BD = BC - CD = 5 - 2 = 3. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \text{在 Rt}\triangle ABD \text{ 中, } AB = \sqrt{BD^2 + AD^2} = 5, \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \sin \angle BAD = \frac{BD}{AB} = \frac{3}{5}. \quad \dots\dots\dots (1 \text{ 分})$$

$$(2) \because AB = BC = 5, BF \text{ 平分 } \angle ABC, \\ \therefore BF \perp AC, \quad AF = \frac{1}{2} AC = \sqrt{5}. \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore \angle AFE = \angle ADC, \text{ 又 } \because \angle EAF = \angle CAD, \therefore \triangle AEF \sim \triangle ACD, \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{EF}{CD} = \frac{AF}{AD}. \text{ 即 } \frac{EF}{2} = \frac{\sqrt{5}}{4}. \therefore EF = \frac{\sqrt{5}}{2}. \quad \dots\dots\dots (2 \text{ 分})$$

$$22. \text{ 解: 设 } OH \text{ 的长为 } x \text{ 米.} \quad \dots\dots\dots (1 \text{ 分})$$

$$\text{在 Rt}\triangle OBH \text{ 中, } \because \tan \angle OBH = \frac{OH}{BH}, \therefore BH = \frac{x}{\tan 63.4^\circ} \approx \frac{x}{2}. \quad \dots\dots\dots (3 \text{ 分})$$

$$\text{在 Rt} \triangle AOH \text{ 中, } \because \tan \angle OAH = \frac{OH}{AH}, \quad \therefore$$

$$AH = \frac{x}{\tan 26.5^\circ} \approx \frac{x}{0.5} = 2x. \quad \dots\dots\dots (3 \text{ 分})$$

$$\because AB = AH - BH, \quad AB = 13, \\ \therefore 2x - \frac{x}{2} = 13. \quad \text{解得 } x = \frac{26}{3} \approx 8.7 \text{ (米)}. \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore \text{路灯顶端 } O \text{ 到地面的距离 } OH \text{ 的长为 } 8.7 \text{ 米.} \quad \dots\dots\dots (1 \text{ 分})$$

$$23. \text{ 证明: (1) } \because AB^2 = AE \cdot AC, \therefore \frac{AE}{AB} = \frac{AB}{AC}. \quad \dots\dots\dots (1 \text{ 分})$$

$$\text{又 } \because \angle BAE = \angle CAB, \therefore \triangle ABE \sim \triangle ACB. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \angle ABF = \angle C, \quad \angle ABC = \angle AEB. \quad \dots\dots\dots (1 \text{ 分})$$

$$\because \angle ABC = \angle AFE, \therefore \angle AFE = \angle AEB. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore 180^\circ - \angle AFE = 180^\circ - \angle AEB, \text{ 即 } \angle AFB = \angle BEC. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \triangle ABF \sim \triangle BCE. \quad \dots\dots\dots (1 \text{ 分})$$

$$(2) \because \triangle ABF \sim \triangle BCE, \therefore \frac{CE}{CB} = \frac{BF}{AB}, \quad \angle CBE = \angle BAF. \quad \dots\dots\dots (2 \text{ 分})$$

$$\text{又 } \because \angle BDF = \angle ADB, \therefore \triangle DBF \sim \triangle DAB. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{BF}{AB} = \frac{DF}{DB}, \quad \therefore \frac{CE}{CB} = \frac{DF}{DB}. \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore DF \cdot BC = DB \cdot CE. \quad \dots\dots\dots (1 \text{ 分})$$

$$24. \text{ 解: (1) 将 } A(-1, 0)、B(2, 0) \text{ 代入 } y = ax^2 + bx + 2, \text{ 得}$$

$$\begin{cases} a-b+2=0 \\ 4a+2b+2=0. \end{cases} \quad \text{解得: } \begin{cases} a=-1 \\ b=1. \end{cases} \quad \dots\dots\dots (2 \text{ 分})$$

所以, $y = -x^2 + x + 2$. $\dots\dots\dots (1 \text{ 分})$

当 $x=0$ 时, $y=2$. $\therefore$ 点 C 的坐标为 $(0, 2)$ $\dots\dots\dots (1 \text{ 分})$

(2) ①过点 P 作 $PH \perp BC$, 垂足为点 H .

$\therefore P(1, m)$ 在 $y = -x^2 + x + 2$ 上,
 $\therefore m = -1 + 1 + 2 = 2$, $P(1, 2)$ $\dots\dots\dots (1 \text{ 分})$

$\therefore C(0, 2)$, $B(2, 0)$,
 $\therefore BC = 2\sqrt{2}$. $PC \perp OC$, $\angle BCO = 45^\circ$, $\angle PCH = 45^\circ$. $\dots\dots\dots (1 \text{ 分})$

$\therefore CH = PH = \frac{PC}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. $BH = BC - CH = 2\sqrt{2} - \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$. $\dots\dots\dots (1 \text{ 分})$

$\therefore \tan \angle PBC = \frac{PH}{BH} = \frac{\frac{\sqrt{2}}{2}}{\frac{3\sqrt{2}}{2}} = \frac{1}{3}$. $\dots\dots\dots (1 \text{ 分})$

②由题意可知, 点 Q 在第二象限. 过点 Q 作 $QD \perp x$ 轴, 垂足为点 D .

$\therefore \angle QBP = \angle CBA = 45^\circ$, $\therefore \angle QBD = \angle CBP$.
 $\therefore \tan \angle PBC = \frac{1}{3}$. $\therefore \tan \angle QBD = \frac{QD}{BD} = \frac{1}{3}$. $\dots\dots\dots (1 \text{ 分})$

设 $DQ = a$, 则 $BD = 3a$, $OD = 3a - 2$. $\therefore Q(2 - 3a, a)$. $\dots\dots\dots (1 \text{ 分})$

将 $Q(2 - 3a, a)$ 代入 $y = -x^2 + x + 2$, 得
 $-(2 - 3a)^2 + 2 - 3a + 2 = a$.

解得 $a_1 = \frac{8}{9}$, $a_2 = 0$ (舍). $\therefore P(-\frac{2}{3}, \frac{8}{9})$. $\dots\dots\dots (2 \text{ 分})$

25. 解: (1) 过 D 作 $DH \perp BC$, 垂足为点 H . $\dots\dots\dots (1 \text{ 分})$

$\therefore \angle C = 90^\circ$, $\therefore DH \parallel AC$. $\therefore \frac{BH}{BD} = \frac{BC}{BA} = \frac{4}{5}$. $\dots\dots\dots (1 \text{ 分})$

$\therefore BD = DE = 5t$, $\therefore BH = EH = 4t$. $\dots\dots\dots (1 \text{ 分})$

又 $\therefore BC = 8$, $CE = 4t$, $\therefore 12t = 8$, $t = \frac{2}{3}$. $\dots\dots\dots (1 \text{ 分})$

(2) 当 $t = \frac{2}{5}$ 时, 得 $BD = 2$, $CE = \frac{8}{5}$, $BE = \frac{32}{5}$.

$\therefore BE > BD$, $\therefore$ 点 F 是射线 ED 与直线 BF 的交点 $\dots\dots\dots (1 \text{ 分})$

过 E 作 $EG \parallel AC$, 交 AB 于点 G , 则 $BF \parallel GE \parallel AC$.

$\therefore \frac{AG}{AB} = \frac{CE}{CB}$, $AG = 2$. $\therefore DG = 10 - 2 - 2 = 6$. $\dots\dots\dots (1 \text{ 分})$

$\therefore \frac{BF}{GE} = \frac{BD}{DG} = \frac{2}{6} = \frac{1}{3}$, $\frac{GE}{AC} = \frac{BG}{BA} = \frac{8}{10} = \frac{4}{5}$. $\dots\dots\dots (1 \text{ 分})$

$$\therefore \frac{BF}{AC} = \frac{BF}{GE} \times \frac{GE}{AC} = \frac{1}{3} \times \frac{4}{5} = \frac{4}{15}. \quad \dots\dots\dots (1 \text{ 分})$$

(3) (i) 当点 F 是射线 ED 与 BF 的交点时,

$\because \angle BDE > \angle F$, $\angle BDE > \angle FBD$, 又 $\because \triangle BDE$ 与 $\triangle BDF$ 相

似,

$$\begin{aligned} \therefore \angle BDE &= \angle BDF = 90^\circ. \quad \because \angle BDE = \angle C, \quad \angle DBE = \angle CBA, \\ \therefore \triangle BDE &\sim \triangle BCA. \quad \dots\dots\dots (1 \text{ 分}) \end{aligned}$$

$$\therefore \frac{BD}{BC} = \frac{BE}{BA}. \quad \text{即 } \frac{5t}{8} = \frac{8-4t}{10}. \quad \text{解得 } t = \frac{32}{41}. \quad \therefore$$

$$BD = \frac{160}{41}. \quad \dots\dots\dots (1 \text{ 分})$$

$$\because \angle F = \angle DBE, \quad \therefore \sin \angle F = \sin \angle DBE. \quad \therefore \frac{BD}{BF} = \frac{AC}{AB}. \quad \text{解得}$$

$$BF = \frac{800}{123}. \quad (1 \text{ 分})$$

(ii) 当点 F 是射线 DE 与 BF 的交点时,

$\because \triangle BDE$ 与 $\triangle BDF$ 相似, 又 $\because \angle BDE = \angle BDF$, $\therefore \angle DBE = \angle F$, 即 $\angle ABC = \angle F$,

又 $\because \angle EBF = \angle C$, $\therefore \triangle BEF \sim \triangle CAB$.

$$\therefore \frac{BF}{BC} = \frac{BE}{AC}, \quad \text{即 } \frac{BF}{8} = \frac{8-4t}{6}. \quad \text{解得 } BF = \frac{4}{3}(8-4t). \quad \dots\dots\dots (1 \text{ 分})$$

过 D 作 $DM \perp BC$, 垂足为点 M . 由 $BD = 5t$, 得 $DM = 3t$, $BM = 4t$, $EM = 8t - 8$.

$$\because \quad \quad \quad BF \parallel DM, \quad \quad \quad \therefore \angle EDM = \angle F = \angle ABC. \quad \therefore \tan \angle EDM = \tan \angle ABC.$$

$$\therefore DM = \frac{4}{3}(8t - 8). \quad \therefore \frac{4}{3}(8t - 8) = 3t. \quad \text{解得 } t = \frac{32}{23}. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore BF = \frac{4}{3}(8 - 4t) = \frac{224}{69}. \quad \dots\dots\dots (1 \text{ 分})$$

综上所述, 当 $\triangle BDE$ 与 $\triangle BDF$ 相似时, BF 的长为 $\frac{800}{123}$ 或

$$\frac{224}{69}.$$