

2022 年上海市青浦区中考数学一模试卷

官方标答

一、选择题:

1. C; 2. C; 3. A; 4. B; 5. D; 6. D.

二、填空题:

7. 9; 8. $\vec{a}+4\vec{b}$; 9. 2:3; 10. 高; 11. $y=x^2-2$; 12. >;

13. 6; 14. $2\sqrt{3}$; 15. 4; 16. $\frac{\sqrt{10}}{10}$; 17. $2-\sqrt{2}$; 18. $y=-3x+3$.

三、解答题:

19. 解: 原式 = $\left| \frac{\sqrt{2}}{2} - 1 \right| + 2 \times \frac{\sqrt{3}}{2} - (\sqrt{3})^0 - \left(\frac{\sqrt{3}}{3} \right)^{-1}$ (4 分)

= $1 - \frac{\sqrt{2}}{2} + \sqrt{3} - 1 - \sqrt{3}$ (4 分)

= $-\frac{\sqrt{2}}{2}$ (2 分)

20. 解: (1) $\because$ 四边形 $ABCD$ 是平行四边形,

$\therefore AD \parallel BC, AD = BC$ (2 分)

$\frac{BC}{ED} = \frac{BF}{DF}$ (1 分)

$\therefore BF = 3DF, \therefore \frac{BF}{DF} = 3$.

$\frac{BC}{ED} = 3$ (1 分)

$\frac{AD}{ED} = 3$.

$\therefore AE : ED = 2$ (1 分)

(2) $\because AE : ED = 2 : 1, \therefore \overrightarrow{DE} = \frac{1}{2} \overrightarrow{EA}$.

$\therefore \overrightarrow{EA} = \vec{b}$,

$\overrightarrow{DE} = \frac{1}{2} \vec{b}$ (1 分)

$\therefore \overrightarrow{CE} = \overrightarrow{DE} - \overrightarrow{DC}$,

$$\therefore \overrightarrow{CE} = \frac{1}{2}\vec{b} - \vec{a} \quad \dots\dots\dots (1 \text{ 分})$$

$$\because AD \parallel BC, \therefore \frac{CF}{CE} = \frac{BF}{BD} \quad \dots\dots\dots (1 \text{ 分})$$

$$\because BF = 3DF, \therefore \frac{BF}{BD} = \frac{3}{4}, \therefore \frac{CF}{CE} = \frac{3}{4}.$$

$$\therefore \overrightarrow{CF} = \frac{3}{4}\overrightarrow{CE} \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \overrightarrow{CF} = \frac{3}{4}\left(\frac{1}{2}\vec{b} - \vec{a}\right) = \frac{3}{8}\vec{b} - \frac{3}{4}\vec{a} \quad \dots\dots\dots (1 \text{ 分})$$

21. 解: (1) $\because$ 过点 A 作 $AH \perp BD$, 垂足为点 H .

$$\because AB = AD, \therefore BH = HD. \quad \dots\dots\dots (1 \text{ 分})$$

$\because$ 点 D 是 BC 的中点, $\therefore BD = CD$.

$$\because BD = 4, \therefore CD = 4.$$

$$\therefore HC = 6. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore \tan C = \frac{1}{4}, \therefore \frac{AH}{HC} = \frac{1}{4}, \therefore AH = \frac{3}{2}. \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore AB = \sqrt{BH^2 + AH^2},$$

$$\therefore AB = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \frac{5}{2}. \quad \dots\dots\dots (2 \text{ 分})$$

(2) 过点 C 作 $CG \perp BA$, 交 BA 的延长线于点 G . $\dots\dots\dots (1 \text{ 分})$

$$\therefore \sin B = \frac{AH}{AB} = \frac{CG}{BC}, \quad \dots\dots\dots (2 \text{ 分})$$

$$\therefore \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{CG}{8} \quad \dots\dots\dots (1 \text{ 分})$$

$$\therefore CG = \frac{24}{5}.$$

$$\therefore \text{点 } C \text{ 到直线 } AB \text{ 的距离为 } \frac{24}{5}. \quad \dots\dots\dots (1 \text{ 分})$$

22. 解: 过点 C 作 $CH \perp BD$, 垂足为点 H . $\dots\dots\dots (1 \text{ 分})$

由题意, 得 $\angle DCH = 27^\circ$, $\angle HCB = 13^\circ$, $AB = CH = 20$ (米).

在 Rt $\triangle DHC$ 中, $\therefore \tan \angle DCH = \frac{DH}{CH}$, $\therefore DH = \tan 27^\circ \times 20 \approx 10.2$ (4 分)

在 Rt $\triangle HCB$ 中, $\therefore \tan \angle HCB = \frac{HB}{CH}$, $\therefore BH = \tan 13^\circ \times 20 \approx 4.6$ (4 分)

$\therefore BD = HD + HB \approx 10.2 + 4.6 = 14.8$ (米). (1 分)

答: 教学楼 BD 的高度约为 14.8 米.

23. 证明: (1) $\therefore DC^2 = DE \cdot DB$,

$$\therefore \frac{DC}{DE} = \frac{DB}{DC}. \dots\dots\dots (1 \text{ 分})$$

又 $\therefore \angle CDE = \angle BDC$, $\therefore \triangle DCE \sim \triangle DBC$ (1 分)

$\therefore \angle DCE = \angle DBC$ (1 分)

$\therefore \angle ABD = \angle DBC$,

$\therefore \angle DCE = \angle ABD$ (1 分)

又 $\therefore \angle AEB = \angle DEC$, $\therefore \triangle AEB \sim \triangle DEC$ (2 分)

$$(2) \therefore \triangle AEB \sim \triangle DEC, \therefore \frac{AE}{EB} = \frac{DE}{EC}. \dots\dots\dots (1 \text{ 分})$$

又 $\therefore \angle AED = \angle BEC$, $\therefore \triangle AED \sim \triangle BEC$ (1 分)

$\therefore \angle ADE = \angle BCE$ (1 分)

又 $\therefore \angle ABD = \angle DBC$, $\therefore \triangle BDA \sim \triangle BCE$ (1 分)

$$\therefore \frac{BD}{BC} = \frac{DA}{CE}. \dots\dots\dots (1 \text{ 分})$$

$$\therefore BC \cdot AD = CE \cdot BD. \dots\dots\dots (1 \text{ 分})$$

24. 解: (1) 将 $A(-1, 0)$ 、 $B(3, 0)$ 代入 $y = x^2 + bx + c$, 得

$$\begin{cases} 1 - b + c = 0, \\ 9 + 3b + c = 0. \end{cases} \quad \text{解得: } \begin{cases} b = -2, \\ c = -3. \end{cases} \dots\dots\dots (2 \text{ 分})$$

所以, $y = x^2 - 2x - 3$ (1 分)

当 $x=0$ 时, $y=-3$. $\therefore$ 点 C 的坐标为 $(0, -3)$ (1 分)

(2) $\therefore y = x^2 - 2x - 3 = (x-1)^2 - 4$, $\therefore$ 点 D 的坐标为 $(1, -4)$ (1 分)

$$\therefore B(3, 0)、C(0, -3)、D(1, -4), \therefore BC = 3\sqrt{2}, DC = \sqrt{2}, BD = 2\sqrt{5}.$$

$$\therefore BC^2 + DC^2 = 18 + 2 = 20 = DB^2. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \angle BCD = 90^\circ. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \tan \angle CBD = \frac{DC}{BC} = \frac{\sqrt{2}}{3\sqrt{2}} = \frac{1}{3}. \dots\dots\dots (1 \text{ 分})$$

$$(3) \because \tan \angle ACO = \frac{AO}{OC} = \frac{1}{3}, \therefore \angle ACO = \angle CBD. \dots\dots\dots (1 \text{ 分})$$

$$\because OC = OB, \therefore \angle OCB = \angle OBC = 45^\circ. \therefore \angle ACO + \angle OCB = \angle CBD + \angle OBC.$$

$$\text{即: } \angle ACB = \angle DBO. \dots\dots\dots (1 \text{ 分})$$

$\therefore$ 当 $\triangle BDP$ 与 $\triangle ABC$ 相似时, 点 P 在点 B 左侧.

$$(i) \text{ 当 } \frac{AC}{CB} = \frac{DB}{BP} \text{ 时,}$$

$$\therefore \frac{\sqrt{10}}{3\sqrt{2}} = \frac{2\sqrt{5}}{BP}. \therefore BP = 6. \therefore P(-3, 0). \dots\dots\dots (1 \text{ 分})$$

$$(ii) \text{ 当 } \frac{AC}{CB} = \frac{BP}{DB} \text{ 时,}$$

$$\therefore \frac{\sqrt{10}}{3\sqrt{2}} = \frac{BP}{2\sqrt{5}}. \therefore BP = \frac{10}{3}. \therefore P(-\frac{10}{3}, 0). \dots\dots\dots (1 \text{ 分})$$

综上, 点 P 的坐标为 $(-3, 0)$ 或 $(-\frac{10}{3}, 0)$.

25. 解: (1) 过点 A 、 D 分别作 $AH \perp BC$ 、 $DG \perp BC$, 垂足分别为点 H 、点 G .

$$\text{可得: } AD = HG = 2, AH = DG. \because \tan \angle ABC = 2, AB = \sqrt{5},$$

$$\therefore AH = 2, BH = 1. \dots\dots\dots (2 \text{ 分})$$

$$\therefore DG = 2.$$

$$\therefore DC = 2\sqrt{5}, \therefore CG = \sqrt{DC^2 - DG^2} = 4. \dots\dots\dots (1 \text{ 分})$$

$$\therefore BC = BH + HG + GC = 1 + 2 + 4 = 7. \dots\dots\dots (1 \text{ 分})$$

(2) 过点 E 作 $EM \perp BC$, 垂足为点 M . 可得 $EM = 2$.

$$\text{由 (1) 得, } \tan \angle C = \frac{DG}{GC} = \frac{1}{2}.$$

$$\because FB = FE, \therefore \angle FEB = \angle FBE.$$

$$\because \angle FEB = \angle C, \therefore \angle FBE = \angle C. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \tan \angle FBE = \frac{1}{2} \therefore \frac{EM}{BM} = \frac{1}{2}, \therefore BM = 4. \dots\dots\dots (1 \text{ 分})$$

$$\therefore FM^2 + EM^2 = FE^2, \therefore (4 - FB)^2 + 2^2 = FB^2 \dots\dots\dots (1 \text{ 分})$$

$$\therefore BF = \frac{5}{2} \dots\dots\dots (1 \text{ 分})$$

(3) 过点 E 作 $EN \parallel DC$, 交 BC 的延长线于点 N .

$\therefore DE \parallel CN$, $\therefore$ 四边形 $DCNE$ 是平行四边形.

$\therefore DE = CN, \angle DCB = \angle ENB$.

$\therefore \angle FEB = \angle DCB, \therefore \angle FEB = \angle ENB. \dots\dots\dots (1 \text{ 分})$

又 $\therefore \angle EBF = \angle NBE$,

$\therefore \triangle BEF \sim \triangle BNE. \dots\dots\dots (1 \text{ 分})$

$$\therefore \frac{BF}{BE} = \frac{BE}{BN} \therefore BE^2 = BF \cdot BN \dots\dots\dots (1 \text{ 分})$$

过点 E 作 $EQ \perp BC$, 垂足为点 Q . 可得 $EQ = 2, BQ = x + 3$.

$$\therefore BE^2 = QE^2 + BQ^2 = (x + 3)^2 + 2^2 = x^2 + 6x + 13 \dots\dots\dots (1 \text{ 分})$$

$$\therefore y(7 + x) = x^2 + 6x + 13$$

$$\therefore y = \frac{x^2 + 6x + 13}{7 + x} \left(0 < x \leq \frac{1 + \sqrt{145}}{2} \right) \dots\dots\dots (2 \text{ 分})$$