

2020 年上海市静安区中考数学一模试卷

参考答案

2020.1

一、选择题

1. C; 2. D; 3. B; 4. A; 5. C; 6. D.

二、填空题

7. $x(x-5)$; 8. $\sqrt{10}$; 9. $x=3$; 10. $\frac{3}{4}$; 11. 4; 12. 16:25; 13. $15\sqrt{3}$;

14. $y = 200(1+x)^2$ 或 $y = 200x^2 + 400x + 200$; 15. 240; 16. $>$; 17. $\frac{2}{3}$; 18. $\frac{1}{7}$.

三、解答题

19. 解: 原式 =

$$\frac{x-y}{x+2y} \cdot \frac{(x+2y)^2}{(x+y)(x-y)} \dots\dots\dots (4 \text{ 分})$$

$$= \frac{x+2y}{x+y} \dots\dots\dots (2 \text{ 分})$$

当 $x=\sin 45^\circ = \frac{\sqrt{2}}{2}$, $y=\cos 60^\circ = \frac{1}{2}$ 时..... (2 分)

$$\text{原式} = \frac{\frac{\sqrt{2}}{2} + 2 \times \frac{1}{2}}{\frac{\sqrt{2}}{2} + \frac{1}{2}} = \sqrt{2} \dots\dots\dots (2 \text{ 分})$$

20. 解: (1) $\because CD \perp AB$, $\therefore \angle ADC = \angle BDC = 90^\circ$,

在 $\text{Rt}\triangle ACD$ 中, $\sin A = \frac{CD}{AC}$, $\therefore CD = AC \cdot \sin A = 20 \times \frac{3}{5} = 12$ (2 分)

$\therefore AD = \sqrt{AC^2 - CD^2} = \sqrt{20^2 - 12^2} = 16$ (1 分)

$\therefore \tan A = \frac{CD}{AD} = \frac{3}{4}$ (1 分)

$\because \angle ACB = 90^\circ$, $\therefore \angle DCB + \angle B = \angle A + \angle B = 90^\circ$, $\therefore \angle DCB = \angle A$ (1 分)

$$\therefore BD = CD \cdot \tan \angle DCB = CD \cdot \tan A = 12 \times \frac{3}{4} = 9. \dots\dots\dots (2 \text{ 分})$$

$$(2) \because AB = AD + DB = 16 + 9 = 25 \quad \therefore \frac{AD}{AB} = \frac{16}{25}. \dots\dots\dots (1 \text{ 分})$$

$$\text{又} \because \overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{BC} = \vec{a} - \vec{b}, \dots\dots\dots (1 \text{ 分})$$

$$\therefore \overrightarrow{AD} = \frac{16}{25} \overrightarrow{AB} = \frac{16}{25} \vec{a} - \frac{16}{25} \vec{b}. \dots\dots\dots (1 \text{ 分})$$

$$21. \text{ 解: (1) } \because \text{对称轴为 } x = -\frac{b}{2}$$

$$\therefore -\frac{b}{2} = 1. \dots\dots\dots (1 \text{ 分})$$

$$\therefore b = -2. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \text{抛物线的表达式为 } y = x^2 - 2x + 1. \dots\dots\dots (1 \text{ 分})$$

$$(2) \because \text{点 } A(8, m) \text{ 在该抛物线的图像上, } \therefore \text{当 } x=8 \text{ 时,}$$

$$y = x^2 - 2x + 1 = (x-1)^2 = (8-1)^2 = 49.$$

$$\therefore \text{点 } A(8, 49). \dots\dots\dots (1 \text{ 分})$$

$$\therefore \text{点 } A(8, 49) \text{ 关于对称轴对称的点 } A' \text{ 的坐标为 } (-6, 49). \dots\dots\dots (2 \text{ 分})$$

(3) 表格正确, 得 2 分; 图正确得 2 分.

$$22. \text{ 解: (1) 过点 } M \text{ 作 } MD \perp AC \text{ 交 } AC \text{ 的延长线于 } D, \text{ 设 } DM=x. \dots\dots\dots (1 \text{ 分})$$

$$\because \text{在 Rt}\triangle CDM \text{ 中, } CD = DM \cdot \tan \angle CMD = x \cdot \tan 22^\circ, \dots\dots\dots (1 \text{ 分})$$

$$\text{又} \because \text{在 Rt}\triangle ADM \text{ 中, } \angle MAC = 45^\circ,$$

$$\therefore AD = DM, \dots\dots\dots (1 \text{ 分})$$

$$\therefore AD = AC + CD = 100 + x \cdot \tan 22^\circ, \dots\dots\dots (1 \text{ 分})$$

$$\therefore 100 + x \cdot \tan 22^\circ = x. \dots\dots\dots (1 \text{ 分})$$

$$\therefore x = \frac{100}{1 - \tan 22^\circ} \approx \frac{100}{1 - 0.404} \approx 167.785 \approx 167.79. \dots\dots\dots (2 \text{ 分})$$

答: 轮船 M 到海岸线 l 的距离约为 167.79 米.

(2) 作 $\angle DMF=30^\circ$, 交 l 于点 F .

在 $\text{Rt}\triangle DMF$ 中, $DF=DM\cdot\tan\angle FMD=DM\cdot\tan30^\circ$

$$=\frac{\sqrt{3}}{3}DM\approx\frac{1.732}{3}\times 167.79\approx 96.87 \text{ 米.} \dots\dots\dots (1 \text{ 分})$$

$$\therefore AF=AC+CD+DF=DM+DF\approx 167.79+96.87=264.66<300. \dots\dots\dots (1 \text{ 分})$$

所以该轮船能行至码头靠岸. $\dots\dots\dots (1 \text{ 分})$

23. 证明: (1) $\because OD^2=OE\cdot OB$,

$$\therefore \frac{OE}{OD}=\frac{OD}{OB}. \dots\dots\dots (1 \text{ 分})$$

$$\because AD\parallel BC, \therefore \frac{OA}{OC}=\frac{OD}{OB}. \dots\dots\dots (2 \text{ 分})$$

$$\therefore \frac{OA}{OC}=\frac{OE}{OD}. \dots\dots\dots (1 \text{ 分})$$

$$\therefore AF\parallel CD. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \text{四边形 } AFCD \text{ 是平行四边形.} \dots\dots\dots (1 \text{ 分})$$

$$(2) \because AF\parallel CD, \therefore \angle AED=\angle BDC, \frac{BE}{BD}=\frac{BF}{BC}. \dots\dots\dots (1 \text{ 分})$$

$$\because BC=BD, \therefore BE=BF, \angle BDC=\angle BCD\dots\dots\dots (1 \text{ 分})$$

$$\therefore \angle AED=\angle BCD.$$

$$\because \angle AEB=180^\circ-\angle AED, \angle ADC=180^\circ-\angle BCD, \therefore \angle AEB=\angle ADC. \dots\dots\dots (1 \text{ 分})$$

$$\because AE\cdot AF=AD\cdot BF, \therefore \frac{AE}{BF}=\frac{AD}{AF}. \dots\dots\dots (1 \text{ 分})$$

$$\because \text{四边形 } AFCD \text{ 是平行四边形, } \therefore AF=CD. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{AE}{BE}=\frac{AD}{DC}. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \triangle ABE\sim\triangle ADC.$$

24. 解: (1) 将 $A(0, -3)$ 、 $B(1, 0)$ 、 $C(3, 0)$ 代入 $y=ax^2+bx+c$ ($a\neq 0$) 得,

$$\begin{cases} 0=a+b-3, \\ 0=9a+3b-4, \\ -3=0+0+c \end{cases} \dots\dots\dots (3 \text{ 分})$$

$$\text{解得} \begin{cases} a = -1, \\ b = 4, \\ c = -3. \end{cases} \therefore \text{此抛物线的表达式是 } y = -x^2 + 4x - 3. \dots\dots\dots (1 \text{ 分})$$

(2) 过点 D 作 $DH \perp BC$ 于 H ,

在 $\triangle ABC$ 中, 设 AC 边上的高为 h , 则

$$S_{\triangle ABD} : S_{\triangle BCD} = (\frac{1}{2} AD \cdot h) : (\frac{1}{2} DC \cdot h) = AD : DC = 3 : 2 \dots\dots\dots (1 \text{ 分})$$

又 $\because DH \parallel y$ 轴,

$$\therefore \frac{CH}{OC} = \frac{DC}{AC} = \frac{DH}{OA} = \frac{2}{5}. \therefore CH = DH = \frac{2}{5} \times 3 = \frac{6}{5}. \dots\dots\dots (1 \text{ 分})$$

$$\therefore BH = BC - CH = 2 - \frac{6}{5} = \frac{4}{5}. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \tan \angle DBC = \frac{DH}{BH} = \frac{3}{2}. \dots\dots\dots (1 \text{ 分})$$

(3) 方法一:

$\because y = -x^2 + 4x - 3 = -(x-2)^2 + 1$, 所以对称轴为直线 $x=2$, 设直线 $x=2$ 与 x 轴交于点 G . $\dots\dots\dots (1 \text{ 分})$

过点 A 作 AF 垂直于直线 $x=2$, 垂足为 F .

$\because OA=OC=3, \angle AOC=90^\circ, \therefore \angle OAC=\angle OCA=45^\circ. \because AF \parallel x$ 轴, $\therefore \angle FAC=\angle OCA=45^\circ.$

$\because AC$ 平分 $\angle BAE, \therefore \angle BAC=\angle EAC$

$\because \angle BAO=\angle OAC-\angle BAC, \angle EAF=\angle FAC-\angle EAC, \therefore \angle BAO=\angle EAF \dots\dots\dots (1 \text{ 分})$

$\because \angle AOB=\angle AFE=90^\circ, \therefore \triangle OAB \sim \triangle FEA, \therefore \frac{OB}{OA} = \frac{EF}{AF} = \frac{1}{3}.$

$\because AF=2, \therefore EF = \frac{2}{3}. \dots\dots\dots (1 \text{ 分})$

$\therefore EG=GF-EF=AO-EF=3-\frac{2}{3}=\frac{7}{3}.$

$$\therefore E \left(2, -\frac{7}{3} \right). \dots\dots\dots (1 \text{ 分})$$

方法二：

延长 AE 至 x 轴，与 x 轴交于点 F ，

$$\because OA=OC=3, \therefore \angle OAC=\angle OCA=45^\circ,$$

$$\because \angle OAB=\angle OAC-\angle BAC=45^\circ-\angle BAC, \angle OFA=\angle OCA-\angle FAC=45^\circ-\angle FAC,$$

$$\because \angle BAC=\angle FAC, \therefore \angle OAB=\angle OFA. \dots\dots\dots (1 \text{ 分})$$

$$\therefore \triangle OAB \sim \triangle OFA, \therefore \frac{OB}{OA} = \frac{OA}{OF} = \frac{1}{3}. \therefore OF=9, \text{ 即 } F(9, 0) \dots\dots\dots (1 \text{ 分})$$

设直线 AF 的解析式为 $y=kx+b$ ($k \neq 0$),

$$\text{可得} \begin{cases} 0 = 9k + b, \\ -3 = b, \end{cases} \text{ 解得} \begin{cases} k = \frac{1}{3}, \\ b = -3, \end{cases} \therefore \text{直线 } AF \text{ 的解析式为 } y = \frac{1}{3}x - 3 \dots\dots\dots (1 \text{ 分})$$

$$\text{将 } x=2 \text{ 代入直线 } AF \text{ 的解析式得 } y = -\frac{7}{3}, \therefore E \left(2, -\frac{7}{3} \right) \dots\dots\dots (1 \text{ 分})$$

25. (1) 与 $\triangle ACD$ 相似的三角形有： $\triangle ABE$ 、 $\triangle ADC$ ，理由如下： $\dots\dots\dots (2 \text{ 分})$

$$\because AB^2 = BE \cdot DC,$$

$$\therefore \frac{BE}{AB} = \frac{AB}{DC}. \dots\dots\dots (1 \text{ 分})$$

$$\because AB=AC, \therefore \angle B=\angle C. \dots\dots\dots (1 \text{ 分})$$

$$\frac{BE}{AB} = \frac{AC}{DC} \dots\dots\dots (1 \text{ 分})$$

$$\therefore \triangle ABE \sim \triangle DCA.$$

$$\because \triangle ABE \sim \triangle DCA, \therefore \angle AED = \angle DAC.$$

$$\because \angle AED = \angle C + \angle EAC, \angle DAC = \angle DAE + \angle EAC, \therefore \angle DAE = \angle C.$$

$$\therefore \triangle ADE \sim \triangle CDA \dots\dots\dots (1 \text{ 分})$$

$$(2) \because \triangle ADE \sim \triangle CDA, \text{ 又 } \because DF \text{ 平分 } \angle ADC, \therefore \frac{DG}{DF} = \frac{DE}{AD} = \frac{AD}{CD} \dots\dots\dots (1 \text{ 分})$$

$$\text{设 } CE=a, \text{ 则 } DE=3CE=3a, CD=4a, \therefore \frac{4a}{AD} = \frac{AD}{4a}, \text{ 解得 } AD=2\sqrt{3}a \text{ (负值已舍)} (2 \text{ 分})$$

$$\therefore \frac{DF}{DG} = \frac{AD}{CD} = \frac{2\sqrt{3}a}{4a} = \frac{\sqrt{3}}{2} \dots\dots\dots (1 \text{ 分})$$

$$(3) \because \angle BAC=90^\circ, AB=AC, \therefore \angle B=\angle C=45^\circ, \therefore \angle DAE=\angle C=45^\circ$$

$$\because DG \perp AE, \therefore \angle DAG=\angle ADF=45^\circ,$$

$$\therefore AG=DG=\frac{\sqrt{2}}{2} AD=\frac{\sqrt{2}}{2} \cdot 2\sqrt{3}a=\sqrt{6}a \dots\dots\dots (1 \text{ 分})$$

$$\therefore EG=\sqrt{DE^2-DG^2}=\sqrt{3}a \dots\dots\dots (1 \text{ 分})$$

$$\because \angle AED=\angle DAC \therefore \triangle ADE \sim \triangle DFA$$

$$\therefore \frac{AD}{DF} = \frac{AE}{AD}, \therefore DF = \frac{AD^2}{AE} = 4(\sqrt{6}-\sqrt{3}) a \dots\dots\dots (1 \text{ 分})$$

$$\therefore \frac{DG}{DF} = \frac{2+\sqrt{2}}{4} \dots\dots\dots (1 \text{ 分})$$