

2022 年上海市浦东新区中考数学一模试卷

官方标答

一、选择题

1. C 2. A 3. D 4. C 5. C 6. C

二、填空题

7. $2\vec{a} + 3\vec{b}$ 8. 30° 9. $y = -x^2 + 8x$ 10. $x = -\frac{1}{2}$
 11. $\frac{3}{5}$ 12. $\frac{4}{9}$ 13. 4 14. 4
 15. $-\vec{a} + \vec{b}$ 16. $\frac{5}{3}$ 17. $\sqrt{2}$ 18. $3\sqrt{7}$

三、解答题

$$19. \tan^2 60^\circ - \frac{\cos 45^\circ}{\cot 45^\circ - \sin 45^\circ} = (\sqrt{3})^2 - \frac{\frac{\sqrt{2}}{2}}{1 - \frac{\sqrt{2}}{2}} = 3 - \frac{\sqrt{2}}{2 - \sqrt{2}} = 3 - (\sqrt{2} - 1) = 2 - \sqrt{2}$$

$$20. (1) \because DE \parallel BC, \therefore \frac{DE}{BC} = \frac{AE}{AC},$$

$$\therefore \frac{2}{3} = \frac{AE}{6}, \therefore AE = 4.$$

$$(2) \because \overrightarrow{BC} = \vec{b} - \vec{a}, \therefore \overrightarrow{ED} = -\frac{2}{3}\overrightarrow{BC} = -\frac{2}{3}(\vec{b} - \vec{a}) = \frac{2}{3}\vec{a} - \frac{2}{3}\vec{b}.$$

21. 过点 C 作 $CH \perp$ 直线 AE , 垂足为 H ,

在 $\text{Rt}\triangle ACH$ 中, $i = \frac{CH}{AH} = \frac{1}{2.4} = \frac{5}{12}$, 设 $CH = 5k$, $AH = 12k$, $AC = 13k$,

$13k = 26$, $k = 2$, 所以 $CH = 10$, $AH = 24$,

在 $\text{Rt}\triangle AED$ 中, $EH = AE + AH = 6 + 24 = 30$, $\tan \angle AED = \frac{DH}{EH}$, $\frac{DH}{EH} \approx 1.11$,

$DH = 33.3$, $DC = DH - CH = 33.4 - 10 = 23.4 \approx 23$,

所以古树 CD 的高度约为 23 米.

$$22. (1) \text{ 在 } \text{Rt}\triangle ABC \text{ 中, } \cos A = \frac{AC}{AB}, \therefore \frac{3}{5} = \frac{6}{AB}, \therefore AB = 10,$$

$$\because AC^2 + BC^2 = AB^2, \therefore BC = 8, \because AD = BD, \therefore CD = \frac{1}{2}AB = 5,$$

$$\therefore CD = BD, \therefore \angle DCB = \angle B, \therefore \cos \angle DCB = \cos B,$$

$$\therefore \frac{CD}{CE} = \frac{BC}{AB}, \therefore \frac{5}{CE} = \frac{8}{10}, \therefore CE = \frac{25}{4}.$$

$$(2) \text{ 过点 } E \text{ 作 } EH \perp AB, \text{ 垂足为 } H, BE = BC - CH = 8 - \frac{25}{4} = \frac{7}{4},$$

$$\therefore \sin B = \frac{EH}{BE}, \therefore EH = \frac{21}{20}, \therefore \sin B = \sin \angle DCB, \therefore DE = \frac{15}{4},$$

$$\therefore \sin \angle BDE = \frac{EH}{DE} = \frac{7}{25}.$$

$$23. (1) \because \angle ADE = \angle DAE, \angle B = \angle ADE, \therefore \triangle BAC \sim \triangle DAE, \therefore \frac{BA}{AC} = \frac{DA}{AE},$$

$$\therefore \angle BAC = \angle DAE, \therefore \angle BAD = \angle CAE, \therefore \triangle BAD \sim \triangle CAE.$$

$$(2) \because \triangle ABD \sim \triangle ACE, \therefore \frac{AD}{BD} = \frac{AE}{EC} = \sqrt{3},$$

$$\text{在 Rt} \triangle ADE \text{ 中, } \angle ADE = 30^\circ, \therefore \frac{AD}{AE} = \sqrt{3}$$

$$\therefore \frac{AD}{CE} = \frac{AD}{AE} \cdot \frac{AE}{EC} = \sqrt{3} \times \sqrt{3} = 3, \therefore \triangle ADF \sim \triangle ECF, \therefore \frac{DF}{CF} = \frac{AD}{EC} = 3.$$

$$23. (1) \text{ 把 } A(-1,0), B(3,0) \text{ 代入 } y = -x^2 + bx + c \text{ 中,}$$

$$\text{得 } b = 2, c = 3, \therefore y = -x^2 + 2x + 3.$$

$$(2) \text{ 点 } C \text{ 坐标 } (0,3),$$

$$\therefore AE = CE, \therefore \sqrt{(t+1)^2} = \sqrt{t^2 + 3^2}, \therefore t = 4. (3) l_{BC}: y = -x + 3,$$

$$\text{过点 } P \text{ 作 } PH \perp y \text{ 轴, 交 } BC \text{ 于点 } H, \text{ 则 } \triangle PQH \text{ 是等腰直角三角形,}$$

$$\text{设点 } P(p, -p^2 + 2p + 3), H(p, -p + 3), PH = -p^2 + 3p = -\left(p - \frac{3}{2}\right)^2 + \frac{9}{4},$$

$$\text{当且仅当 } p = \frac{3}{2} \text{ 时, } PH \text{ 的最大值是 } \frac{9}{4}, PQ = \frac{\sqrt{2}}{2} PH = \frac{9}{8} \sqrt{2},$$

$$\text{当点 } P\left(\frac{3}{2}, \frac{15}{4}\right) \text{ 时, } PQ \text{ 的最大值是 } \frac{9}{8} \sqrt{2}.$$

$$25. (1) \because OE = OD, \therefore \angle ODE = \angle OED, \because OD \perp AB, EP \perp ED, \therefore \angle ADO = \angle PED,$$

$$\therefore \angle ADO + \angle ODE = \angle PED + \angle OED, \therefore \angle ADE = \angle AEP,$$

$$\therefore \angle A = \angle A, \therefore \triangle ADE \sim \triangle AEP. (2) \because OD \perp AP, BC \perp AB, \therefore OD \parallel BC,$$

$$\therefore \frac{AD}{AO} = \frac{AB}{AC}, \therefore AD = \frac{4}{5}x, DO = EO = \frac{3}{5}x,$$

$$\because AE^2 = AD \cdot AP, \therefore \left(x + \frac{3}{5}x\right)^2 = \frac{4}{5}x \cdot y, \therefore y = \frac{16}{5}x \left(0 < x \leq \frac{25}{8}\right).$$

$$(3) 1^\circ \text{ 当点 } P \text{ 在线段 } AB \text{ 上时, } BP = 4 - y = 4 - \frac{16}{5}x,$$

$$\because \triangle PBF \sim \triangle PED, \therefore \frac{BF}{BP} = \frac{ED}{EP},$$

$$\because \triangle ADE \sim \triangle AEP, \therefore \frac{AE}{AP} = \frac{ED}{EP}, \therefore \frac{BF}{BP} = \frac{AE}{AP},$$

$$\therefore \frac{1}{4 - \frac{16}{5}x} = \frac{\frac{8}{5}x}{\frac{16}{5}x}, \therefore x = \frac{5}{8}, \therefore AP = 2.$$

2° 当点 P 在 AB 延长线上时,

$$\because \angle CFE = \angle PFB = \angle PDE, \angle CEF + \angle DEO = \angle PDE + \angle EDO,$$

$$\therefore \angle CEF = \angle PDE, \therefore \angle CEF = \angle CFE, \therefore EC = FC = 2.$$

过点 E 作 $EG \perp CF$, 垂足是点 G ,

$$\therefore \frac{EG}{CE} = \frac{AB}{AC}, \therefore EG = \frac{8}{5}, CG = \frac{6}{5}, \because EG \parallel BP, \therefore \frac{EG}{PB} = \frac{GF}{BF},$$

$$\therefore PB = 2, \therefore AP = 2 + 4 = 6. \text{ 综上所述, } AP = 2 \text{ 或 } 6.$$